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Articles

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Do Captive Insurance Subsidiaries Improve Cash Flow? Evidence from Nasdaq-100 Companies

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Abstract

This study examines the impact of establishing a captive insurance subsidiary on its parent company's cash flow, using a sample of the 2020 Nasdaq-100 index constituents. This index stands out due to its composition of large-cap, non-financial, and technology-oriented stocks. Forming a captive is different from buying commercial insurance from the standpoints of risk retention and risk transfer, while the efficiency of risk management via captives suggests they have the potential to improve cash flow. Three crucial insights emerge. First, a captive structure is not commonly implemented by Nasdaq-100 companies, as only 15 of them form captives, in contrast to around a third of S&P 500 companies. Second, our multivariate analysis finds no positive relationship between captive formations and cash flow ratios over the entire sample period 1995–2020. This implies that the formation of a captive does not necessarily lead to higher cash flow. Third, our disaggregated datasets reveal that a captive vehicle conditionally improves cash flow when its parent is a relatively young publicly traded entity, carries higher cash holdings, or is affiliated with Consumer Staples, Information Technology, and Communication Services. In addition, our work shows a positive link between captive utilization and cash flow during the post-financial crisis period 2009–2020, an outcome that reflects changes in captive operations following the financial crisis.

1. Introduction

The Nasdaq stock exchange is well known as the home of numerous prominent technology companies. The Nasdaq-100 index (hereafter, "Index") distinguishes itself from the S&P 500 index because of high proportion of technology sector constituents and its stellar performance. According to a report by Nasdaq (2020), the Nasdaq-100 index derives 57 percent of its weight from the technology sector, while the broad-market S&P 500 index draws 23 percent of its weight from this sector.[1] In addition, the former provided an annualized return of almost 15 percent, compared to around 8 percent from the latter over the years from 2007 to 2020.[2] The outsize performance of this Index raises a question whether large-cap Nasdaq companies use captive insurance subsidiaries/companies (hereafter, "captives") as much as S&P 500 companies observed by the prior studies, and whether the use of captives has any impact on cash flow.[3]

A captive insurance structure is a more sophisticated and lesser-known vehicle of risk financing —an alternative technique to traditional commercial insurance solutions. In other words, one parent company can insure its risks through the formation of a pure captive insurance subsidiary under its corporate umbrella, instead of transferring risk to third-party carriers by paying premiums externally.[4] The choice between alternative and conventional risk-financing techniques has a lot to do with a firm's risk appetite. Generally speaking, firms turn to captive insurance mainly because of easier access to reinsurance markets as well as better coverage availability for certain insurable risks[5] (Rejda & McNamara, 2020; Culp, 2006). Moreover, a captive structure allows companies to put

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[1] Both the Nasdaq-100 and S&P 500 indexes are market value-weighted. Thus, the percentage may vary day by day.

[2] The Nasdaq-100 index has attained cumulative total returns of about 2.5 times that of S&P 500 Index.

[3] Previous studies on the use of captives have looked at the broader indexes (Chang et al., 2020; Chang & Chen, 2018, 2019). Nevertheless, their work does not examine the relationship between captives and cash flow.

[4] Captives can be formed on a group basis. That is a group or an association captive owned by a number of parent companies. As a result, it is also known as a member-owned captive. See more discussions in Born (2021).

[5] Not all risks faced by the parent are insurable.

into practice mutual insurance concepts of risk sharing through the alignment of interests of the insured and insurer. On the surface, a captive operates at a smaller scale than typical mutual insurance companies that can insure participants in the commercial marketplace (Chang, Chen, & Weston, 2021; FERMA, 2017). As a matter of fact, a captive can set rates based on specific loss experience of an individual company that accurately reflect a company's unique risk profile (Willis Towers Watson, 2020). Captives have proliferated in the U.S. since the mid-1990s (Chang & Chen, 2018; Marsh, 2017; Cole & McCullough, 2008).[6] In retrospect, many captives that exist today were formed at times when the tax benefit was a key driving force in captive formations in the late 1990s (Scordis & Porat, 1998; Lai & Witt, 1995; Han & Lai, 1991; Cross et al., 1988; Smith, 1986; Hofflander & Nye, 1984).[7] However, the tax advantage is less important for captive utilization in the 21st century.[8]

Additional reasons for the use of captives have developed or been strengthened in the past two decades. In particular, the efficiency of risk management adds to the appeal of arranging a wholly-owned captive which can work as an effective tool to manage risk and maximize value (Aon, 2021; Marsh, 2021, 2017; FERMA, 2017; Willis Towers Watson, 2017; CICA, 2016; Colaizzo, 2009; Holzheu et al., 2003; Petroni, 1998). There is little doubt that managers have carried out risk management activities to their advantage based on their companies' best interests and bottom lines in hopes of adding value to their firms (Segal, 2011). It is also worth noting that captive

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[6] According to the World Domicile Update 2020, published by *Captive Review*, there are 6,304 captives worldwide by the end of 2020, while more than half of them are domiciled in North America.

[7] According to Marsh (2017), in the 1960s, changes in the U.S. tax code, along with other international legislative changes, laid the groundwork for the revolutionary growth in captive formation. For example, Ford Motor Co. is one of the early pioneers that saw an opportunity of managing corporate loss exposures effectively via a captive and formed its own in 1967, according to Chang & Chen (2018).

[8] We don't suggest that the tax benefit plays no role at all because the original tax benefits may remain for some firms that are suited to using a captive. According to Marsh (2017) and Holzheu et al. (2003), the tax benefits of using captives have dropped substantially since the early 2000s. The report by CICA (2016) does not include tax benefits as a key factor driving captive formation.

insurance has a potential edge over conventional commercial insurance when it comes to improved cash flow with recaptured premiums and increased control over their risk-financing programs (Born, 2021; Marsh, 2021; Zurich, 2019; Bird, 2016; Westover, 2016). With a captive subsidiary in place, a parent company can implement an integrated approach that streamlines its risk management efforts for a broad range of risks that face the entire enterprise.[9]

The use of captives may expand the opportunity set for firms that seek alternatives for at least some of their insurable exposures. Although the use of captives has increased since the 1990s, some industries have made greater use of captives than others. According to Marsh (2017), the financial services sector has been especially active in utilizing captives. Most non-financial sectors have lagged in alternative risk management practices (Segal, 2011). However, increased expertise in risk management over time has expanded the use of captives in risk financing across all industries. Several tech giants, including Intel in 2003, Apple in 2008, and Alphabet in 2010 began to employ captive insurance as a tool to finance insurable risk. As more technology companies put captives at the core of their risk management strategy in practice, academic research follows to understand the empirical circumstances under which captive insurance may be utilized by non-financial companies, particularly tech companies, to tackle their loss exposures and whether captives can help firms improve cash flow.

Our study aims to examine whether a captive structure can serve to improve cash flow for its parent company, using a sample of companies included in the Nasdaq-100 index in 2020.[10] The results

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[9] Enterprise Risk Management (ERM) is an approach that requires comprehensive risk analysis and encompasses insurance and non-insurance techniques to treat all measurable sources of uncertainty, both financial and non-financial.

[10] The term *cash flow* is referred to as operating cash flow throughout this paper, unless otherwise specified. The computation of operating cash flow considers a firm's principal business activities, including the firm's net income, depreciation and amortization expense, and working capital accounts during a period of time (i.e., a year in this article). To avoid any potential confusion between cash flow and cash, the term *cash* is referred to as cash balances (or cash holdings) on a firm's balance sheet at a given point

merit attention because financial innovation can help firms operate with greater efficiency and maximize value. Depending on circumstances and strategy, captives may be used to retain insured losses more cost effectively. This research may be able to detect some of the factors that may drive captive formation.

We study the effects of captive formation on the parent company's cash flow. The first step in our analysis investigates the difference between firms with and without captives in a univariate setting. The former do have higher levels of cash flow than the latter. Nevertheless, this outcome is not completely consistent when the analysis is performed using the maximum likelihood treatment effects model that controls for firm characteristics and the potential endogeneity concerns of captive utilization in a multivariate setting. In other words, no evidence has been discovered to generally support the hypothesis that captives improve cash flow over the entire sample period 1995–2020 for the 2020 Nasdaq-100 index components. However, further analysis based on various subsamples shows a positive relationship between captive formation and cash flow levels in the following three conditions. First, we find a positive relationship exists during the post-financial crisis period (2009–2020). That is, the time period used for the study matters to the conclusions. Second, we find that relatively young companies with less than 18 years as public firms appear to better leverage their captives to improve cash flow. The implication is that the cost efficiency of using a captive technique facilitates improved cash flow, if a captive structure fits into a parent's risk tolerance. Third, a positive relationship also exists between cash flow and companies with higher cash holdings as well as those companies affiliated with Consumer Staples, Information Technology, and Communication Services. In other words, firms are more capable of generating more cash flow when they have more cash on hand; firms affiliated with these specific industries are more likely to improve cash flow because their risk profiles appear to match the efficiency of risk management via captives. To put it simply, our analysis does not generate evidence of a consistent positive relationship between captive formation and cash flow during the entire period 1995–2020. Nevertheless, a positive link exists at some subgroup levels by time interval, firm age, cash holdings, and industry affiliation.

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in time. The definitions of these two terms is detailed in Table 1.

As technology takes center stage in the 21st century, this study examines the reasons why some Nasdaq-100 companies gravitate to alternative risk transfer via captives, making contributions to the literature in two ways. First, our study expands previous efforts in the literature that analyze whether captive formation provides value in the forms of stock reaction and price-based measures (Chang & Chen, 2019; Cross et al., 1986; Diallo & Kim, 1989). Second, our work on the relationship between captive use and improved cash flow is closely related to the strand that investigates the determinants of captive insurance (Chang et al., 2021; Chang & Chen, 2018). The results can complement existing studies based on S&P 500 companies by providing an empirical example of how non-financial, tech-savvy companies manage their loss exposures via an innovative risk-financing approach. This paper can serve as a stepping stone to better understand the relationship between captive utilization and cash flow in the U.S., particularly in the large-cap, non-financial, and technology-oriented sectors.

The rest of the paper is broken down into four sections. In Section 2, we explain the reasons that this Index is selected for our study and present a testable hypothesis. Section 3 describes the method, data, and sample used for analysis. Section 4 provides statistical results based on the full sample, subsamples, and a survival dataset for robustness testing purposes. Lastly, Section 5 concludes, proposes avenues for future studies, and addresses the limitations of this research.

2. The Nasdaq-100 Index and Hypothetical Development

The Nasdaq-100 index is differentiated from the S&P 500 index—commonly regarded as the foremost gauge of large-cap U.S. equities—in several ways. First, the former is much more weighted toward the technology industry than the latter (Nasdaq, 2020).[11]

[11] The Nasdaq-100 index made its debut in 1985, while the S&P 500 index was launched in 1957. Sources: https://www.nasdaq.com/nasdaq-100 and http://us.spindices.com/indices/equity/sp-500. With its technology-orientated reputation, the Nasdaq-100 index is heavily allocated towards three key industries: Technology, Consumer Services, and Healthcare. This index features some of iconic brands, including Apple, Google, Intel, and

Second, the former outperformed the latter by a wide margin between 2007 and 2020. The annualized return for the Nasdaq-100 was 14.7 percent, in comparison with 8.4 percent for the S&P 500. Third, the former differs from the latter when it comes to constituent selection. The Nasdaq-100 is made up of 100 of the largest domestic and international non-financial companies listed on the Nasdaq Stock Market based on market capitalization. In contrast, the S&P 500 index selects its constituents based on size, sector, and style of stocks trading on U.S. exchanges, and it is widely considered a benchmark to measure the performance of the overall U.S. equity market.

Because technology is interwoven into every aspect of the economy and keeps changing the nature of modern businesses in all sectors, we set out to determine whether non-financial Nasdaq-100 companies are able to boost their cash flow as a result of captive formation.

Hypothesis: The use of captives is positively related to the ratio of cash flow to assets.

The connection between captive insurance and improved cash flow has been well documented in the literature (Marsh, 2021; Born, 2021; Zurich, 2019; Bird, 2016; Westover, 2016). A captive structure sets the stage for the parent company to retain insurance premiums that would otherwise be paid externally to third-party insurers for commercial insurance. The benefit of improved cash flow can be achieved through these recaptured premiums that can be invested by the parent company to maximize returns. In addition, the efficiency of risk management via captives may further help firms allocate internal funds strategically and enhance cash flow management to deliver financial solutions that maximize value (Marsh, 2017, 2019; Chang & Chen, 2018; FERMA, 2017).[12] As a result, captive formation may serve as a predictor of higher cash flow for its parent company.

Tesla, which are usually considered preeminently innovative on the forefront of modern-day technology progress. In comparison, the S&P 500 index consists of components in eleven sectors, and more than three quarters of its market weight is represented by the following five sectors: Information Technology, Health Care, Consumer Discretionary, Financials, and Communication Services.

[12] According to Chang & Chen (2018), cash reserves stored in a captive help

3. Data, Sample, and Method

3.1. Data and Sample

This empirical study is aimed at examining whether the use of captives helps improve cash flow among the constituent companies of the Nasdaq-100 index. Our first step of data collection starts with the 100 components of the index in 2020 because of the two concerns: (1) The index presents itself as one of the world's large-cap growth indexes, constantly adjusting its composition based on market capitalization. As a result, some companies may be effectively included for a short period of time and get removed due to poor market performance. One of several examples is TIME WARNER INC that was included in the index in Nov. 1995, but removed from the list in Aug. 1996. (2) It is common to see mergers and acquisitions in the marketplace, not to mention in the technology sector. For instance, Whole Foods Market Inc (WFM) was included in the index between 2002 and 2008, and was later acquired by Amazon.com—a component of the index in 2020. Thus, our sample consists of Amazon.com only, rather than both of them. As a result, our analysis can generate more relevant results based on current index constituents. The second step is to obtain financial data from COMPUSTAT database and data on public offering dates from CRSP between 1995 and 2020.[13] We then excluded those companies with missing data on basic accounting variables and stock prices, and all continuous variables were winsorized at the 1 percent and 99 percent levels to mitigate the impact of extreme outliers on our analysis. The final step is to consolidate the captive insurance data gathered from the Captive Review's Captive Insurance Database (CID). The final panel dataset is composed of 99 companies with 1,845 firm-year observations from 1995 to 2020.[14]

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the parent company shield the cash for various risk management needs and reduce the demand for distributing cash dividends from its shareholders. The existence of a captive mechanism heralds the strategic use of capital by the parent company (Bodnaruk et al., 2016).

[13] The earliest "effective from date" of Nasdaq-100 companies on COMPUSTAT is 1/1/1995.

[14] Only one of the initial 100 companies was removed in the final dataset because of missing values in the regression dependent and independent variables. Not every company in the sample has the full 1995–2020 data. In addition, the 1994 sale information has been included to calculate the sales growth rate for 1995.

3.2. Univariate Analysis

Given the primary objective of our empirical analysis is to explore the connection between captive insurance and cash flow, we start off with examining how the captive variable is related to other variables in a univariate setting. *Captive* is a binary dummy that indicates whether a firm owns a captive in any given year during the period 1995–2020. That is, *Captive* takes the value of one for firm-years starting from the year when a firm formed a captive and zero before that year. Thus, a firm that forms a captive in 2008 is assigned *Captive* = 0 for firm-years 1995–2007 and *Captive* = 1 for firm-years 2008–2020.

We have tabulated the definitions of all variables in Table 1 and provided in Table 2 the breakdown of active captives by year, by ownership type, and by industry, respectively. The descriptive statistics of our data are illustrated in Table 3, while comparison of means between firms with and without captives is made in Table 4.

Some distinctive features appear as we break down captives formed by Nasdaq-100 companies by year and by type, observed in Table 2a. In general, there are only 15 out of the 100 index constituents that take an alternative approach of using captives for risk financing strategies, and 12 out of these 15 captives (i.e., 80 percent) are formed after the late 1990s. It is also notable that some big tech giants—Microsoft (1998), Intel (2003), Apple (2008), and Alphabet (2010)—have gradually put in place a captive structure for their risk management programs. Furthermore, all captives are structured with pure ownership, and established only to cover the risks of their parent companies. When it comes to captives by industry shown in Table 2b, almost three quarters of captives are formed by companies affiliated with the following sectors: Consumer Staples, Information Technology, and Communication Services.

Table 1: Variable Definitions

Variable	Definition
Captive	= 1 if a firm has a captive insurance subsidiary in a given year and 0 otherwise
CF (Cash Flow)	Net cash flow from operating activities less cash flow from extraordinary items and discontinued operations (Nallareddy et al., 2020). [Compustat: OANCF – XIDOC].
CFR (Cash Flow Ratio)	Ratio of CF to total assets = (OANCF – XIDOC)/AT
Size	Ln (assets) [Compustat: Ln (AT)]
Cash	Cash/total assets [Compustat: CH/AT]
Opacity	Intangible assets/total assets [Compustat: INTAN/AT]
Capex	Capital expenditure/total assets [Compustat: CAPX/AT]
ROA	Net income/total assets [Compustat: NICON/AT]
Sales growth	The percentage growth in annual sales (REVT) from the prior year to the current year [Compustat: REVT]
Dividend	= 1 if a firm paid dividends (DVT) in a given year and 0 otherwise [Compustat: DVT]
Leverage	Book value of long-term debt/Market value of equity [Compustat: DLTT/(PRCCD × CSHOC)]
Age	The number of years since a firm has stock price information in COMPUSTAT Global database

Note: The sources of data include the Captive Insurance Database (CID) for the captive variable and the COMPUSTAT Global database for the rest variables. NA means that no priors on the sign of that variable are expected. All continuous variables are winsorized at the 1% and 99% levels.

Table 2a: Number of Active Captives Formed by Year and by Type

Year \ Type	Pure[a]	Other[b]	Total	Percent	Parent company with a captive (the year its captive was licensed)
1970–1974	0	0	0	0	
1975–1979[c]	2	0	2	13.3	MARRIOTT INTL INC (1977) PEPSICO INC (1978)
1980–1984	0	0	0	0	
1985–1989	1	0	1	6.7	COMCAST CORP (1987)
1990–1994	0	0	0	0	
1995–1999	1	0	1	6.7	MICROSOFT CORP (1998)
2000–2004	5	0	5	33.3	T-MOBILE (2001), PAYCHEX (2003) INTEL (2003), CSX (2004) KRAFT HEINZ (2004)
2005–2009	3	0	3	20	STARBUCKS (2006), APPLE (2008) COSTCO WHOLESALE (2009)
2010–2014	2	0	2	13.3	ALPHABET (2010), WALGREENS (2011)
2015–2020	1	0	1	6.7	REGENERON PHARMACEUTICALS (2016)
Total	15	0	15	100%	

Note: This table exhibits the number of active captives formed by Nasdaq-100 index constituents in 2020. Throughout this study, a captive is referred to as any active captive insurance company in the types of pure, group, cell, special purpose vehicle (SPV), and unknown (NA) ownership. [a]A pure captive is an insurance company owned by one parent company and formed to insure the risks of its parent. [b]Other captives refer to those formed in the types other than pure ownership, such as group, cell, SPV, etc. [c]The PACCAR INC's captive licensed in 1977 is currently dormant, and it is not considered active. Captives in dormant status can buy insurance from the traditional market but return to the captive when the market fluctuates. Therefore, our analysis treats a firm with a dormant captive as a firm without an active captive.

Table 2b: Breakdown of Active Captives by Industry

GIC	Industry	No. of Captives	Proportion of Active Captives
10	Energy	0	0
15	Materials	0	0
20	Industrials	1	6.7
25	Consumer Discretionary	2	13.3
30	Consumer Staples	4	26.7
35	Health Care	1	6.7
40	Financials	0	0
45	Information Technology	4	26.7
50	Communication	3	20.0
55	Utilities	0	0
Total		15	100%

Note: This breakdown uses the two-digit Global Industry Classification (GIC) Standard codes.

According to Table 3, approximately twelve percent of all firm-years are made up of firms with captives. This finding looks pale in comparison with about a third of the firm-year observations based on S&P 500 companies and S&P Europe 350 reported by Chang & Chen (2019) and Chang et al. (2020), respectively. Overall, Nasdaq-100 companies do not embrace the use of captives as prevalently as those large-cap companies included in the S&P 500 and S&P Europe 350 indexes. On average, firms have a 6.4 percent ROA and hold around 16 percent of assets in cash, while almost 19 percent of their assets are intangible.

Table 3: Descriptive Statistics

Variable	*N*	Min	Max	Mean	Median	Std. deviation
Captive	1845	0	1	0.1230	0	0.3285
CF	1845	-284.487	36314	2584.11	730.183	5492.82
CFR	1845	-0.329	0.393	0.128	0.136	0.118
Size	1845	17.555	26.007	22.331	22.399	1.849
Cash	1845	0.002	0.709	0.161	0.124	0.143
Opacity	1845	0	0.773	0.189	0.110	0.210
Capex	1845	0.003	0.214	0.049	0.035	0.043
ROA	1845	-0.469	0.319	0.064	0.082	0.129
Sales growth	1845	-0.955	4.193	0.237	0.121	0.587
Dividend	1845	0	1	0.4	0	0.490
Leverage	1845	0	1.314	0.130	0.044	0.224
Age	1845	0	86	22.067	18	17.868

Note: *Captive* is a dummy variable that equals 1 for a firm-year in which a captive is used and 0 otherwise.

Table 4: Mean Comparison of Nasdaq-100 Companies with and without Captives

Variable	*N*	Firms with captives	*N*	Firms without captives	Mean difference	*t* statistic
CF	227	9637.48	1618	1594.54	8042.94	11.22***
CFR	227	0.15	1618	0.12	0.02	4.69***
Size	227	24.31	1618	22.05	2.25	25.46***
Cash	227	0.07	1618	0.17	-0.10	-19.79***
Opacity	227	0.22	1618	0.18	0.04	2.93***
Capex	227	0.05	1618	0.04	0.01	1.93*
ROA	227	0.09	1618	0.06	0.03	6.94***
Sales growth	227	0.13	1618	0.25	-0.12	-3.90***
Dividend	227	0.84	1618	0.33	0.50	18.64***
Leverage	227	0.17	1618	0.12	0.05	3.591***
Age	227	37.30	1618	19.92	17.37	10.99***

Note: The *p* value is based on a *t* test on the difference in means that assumes unequal variances. ***p<0.01, **p<0.05, *p<0.1.

As seen in Table 4, mean comparison tests between firms that did form captives and did not form captives provide some interesting findings. First, some results are consistent with the prior studies that firms with captives are larger in size and have a lengthier period of operating as publicly traded businesses. Some are different because Nasdaq-100 companies with captives have possessed higher proportions of intangible assets and capital expenditures than their counterparts without captives. Second, the difference is significant and supports our hypothesis that firms with captives are positively related to cash flow ratios.

3.3. Multivariate Analysis

Our analysis employs a treatment effects model to explore the relationship between cash flow and captive use, following Bodnaruk et al. (2016), Hoyt & Liebenberg (2011), and Heckman (1976, 1978). According to Nallareddy et al. (2020), cash flow is defined as net cash flow from operating activities less cash flow from extraordinary items and discontinued operations. To test our hypothesis, the regression has the dependent variable of cash flow ratio (CFR)—the percentage of cash flow in total assets. Captive formation is treated as the variable of interest in the model, along with several control variables. Below are the equations that formulate our regression model.

$$Y_i = \alpha + \beta\, Captive_i + \textstyle\sum_i \lambda_i\, Control\ variable_i + \varepsilon_i \qquad (1)$$

$$Captive_i^* = \delta\omega_i + \mu_i \qquad (2)$$

$$Captive_i = \begin{cases} 1 \text{ if} Captive_i^* > 0 \\ 0 \text{ otherwise.} \end{cases} \qquad (3)$$

where $Captive_i$ in the first equation is an endogenous dummy variable, indicating whether the captive treatment is received, and it is estimated with $Captive_i^*$ from the second equation. That is, $Captive_i^*$ is an unobservable latent variable and a linear function of the coefficient vector ω_i that contains a set of characteristics that affect a firm's choice to form a captive. That is, the second equation reflects the decision to receive the treatment. The observed decision to form a captive in a particular year is expressed in the third equation.

To explain differences in CFR, we incorporate a vector of control variables in Eq. (1): firm size, opacity, capital expenditures, return on assets, sales growth, dividend, leverage, and age.[15] Given the results of Chang & Chen (2018, 2019) and Chang et al. (2021), the determinants of captive formation in Eq. (2) include firm size, cash,

[15] Leverage is used to capture a firm's use of capital, while sales growth is indicative of revenues change year by year.

opacity, capital expenditures, dividend, leverage, and age.[16] The explanatory variables, hypothesized to be determinants of a firm's choice to have a captive, are estimated to generate with $Captive_i^*$. Both equations (1) and (2) are estimated jointly by means of the maximum-likelihood estimation. Consequently, this two-equation model facilitates our investigation into the decision to use captives and the effect of that decision on cash flow. In addition, the equation also controls for time and industry fixed effects; standard errors are adjusted for clustering at the firm level.[17]

It's particularly noteworthy that the benefit of using a treatment-effects model is to better control for a selection bias due to the likely endogeneity of captive decision.[18] This bias arises if we simply model

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[16] Firm size is well documented to be an overriding factor for firms that retain risk, and larger firms are more likely to have captives because of the benefits of risk pooling and scale economy. The existent studies show that the use of captives goes hand in hand with lower levels of cash holdings among S&P 500 firms (Chang & Chen, 2018, 2019). The decision to establish a captive may be attributable to a firm's investment strategy and asset structure. That is, firms that heavily invest in capital expenditures and hold intangible assets are less likely to form a captive because of financial constraints and asset opacity (Chang & Chen, 2018, 2019; Pagach & Warr, 2011). With an improved understanding of its loss experiences over time, a firm with a longer time period of operation is more likely to retain risk with confidence.
[17] All regressions throughout this article included the use of time and industry fixed effects dummy variables, along with firm-level clustering for standard errors, unless otherwise specified. Industry dummies are based on the two-digit Global Industry Classification (GIC) Standard codes.
[18] According to Hoyt & Liebenberg (2011), the treatment-effects model facilitates the adjustment of standard errors for firm-level clustering. In addition, Petersen (2009) suggests that the significance of coefficient estimates will be overstated if the model fails to correct for firm-level clustering issues. Because we have up to 26 repeated observations per firm over the sample period 1995–2020, we must deal with standard errors for clustering by using a maximum-likelihood treatment-effects model. Thus, our results can avoid underestimating the standard errors of our coefficient estimates. As Wooldridge (2010) suggests, all exogenous control variables should be included in both equations of a treatment-effects model. In addition, an instrumental variable in a treatment-effects model is better estimated, especially when all the control variables are firm-level financial statement variables that are likely endogenous (Lennox, Francis, & Wang, 2012).

CFR as a function of a captive dummy and other control variables. In other words, some of the factors that are correlated with the firm's decision to form a captive may also be correlated with observed differences in CFR. To solve the potential endogeneity bias, we employ a maximum-likelihood treatment effects model that simultaneously estimates the decision to form a captive and the effect of that decision on CFR in a two-equation system.

4. Empirical Results

4.1. Captives and Cash Flow Ratios

Our hypothesis asserts that captives may help firms improve cash flow. A captive subsidiary is formed mainly to insure the loss exposures of its parent company. Because premiums are internalized and recaptured via a captive subsidiary instead of flowing out to third-party insurers for commercial insurance, the parent company may streamline all risk management programs under its corporate umbrella and enhance cash flow.

Table 5 presents our estimates on how the use of captives affects cash flow. According to Specification (1) based on the entire sample, the relationship between cash flow ratios and firms with captives is significant and negative. This result from multivariate analysis contradicts against our hypothesis and differs from the finding in the univariate setting (presented in the previous section) that firms with captives have higher levels of cash flow than their counterparts without captives. A plausible justification is that captives are not formed simply as a means of improving a firm's cash flow. Instead, a captive insurance subsidiary often is established to help firms adapt to various risk management needs and insurance market cycles.

Table 5: Captive Insurance and Cash Flow Ratio – Full Sample

Variable	Full sample		Full sample	
	(1)	(2)	(3)	(4)
Panel A: CFR (Eq. 1)				
Captive	-0.160***	-0.162***	-0.075***	-0.0776***
	(0.000)	(0.000)	(0.000)	(0.000)
Size	0.020***	0.0232***	0.005**	0.006**
	(0.002)	(0.001)	(0.035)	(0.017)
Opacity		-0.0427	-0.013	0.001
		(0.186)	(0.384)	(0.975)
Capex		0.595***	0.441***	0.475***
		(0.003)	(0.000)	(0.000)
ROA			0.699***	0.671***
			(0.000)	(0.000)
Sales growth			0.003	0.002
			(0.487)	(0.638)
Dividend				0.007
				(0.315)
Leverage				-0.053***
				(0.000)
Age				-0.000
				(0.912)
Panel B: Captive (Eq. 2)				
Size	0.377***	0.424***	0.488***	0.499***
	(0.000)	(0.000)	(0.000)	(0.000)
Cash	-4.717***	-4.619***	-5.809***	-6.096***
	(0.000)	(0.000)	(0.000)	(0.000)
Opacity	0.051	-0.219	-0.321	-0.276
	(0.859)	(0.601)	(0.593)	(0.635)
Capex	-2.808*	1.246	2.890	3.156
	(0.078)	(0.595)	(0.366)	(0.304)
Dividend	0.500***	0.491***	0.662***	0.720***
	(0.000)	(0.000)	(0.005)	(0.006)
Leverage	0.241	0.134	-0.887*	-1.273***
	(0.367)	(0.648)	(0.055)	(0.004)
Age	-0.005	-0.005	-0.001	-0.001
	(0.121)	(0.138)	(0.934)	(0.928)
No. of observations	1845	1845	1845	1845
No. of clusters	99	99	99	99
Log pseudolikelihood	1160	1208	2055	2073
Wald test of independent equations	68.15***	68.89***	55.96***	63.11 ***

Note: The results of treatment effects are based on 2020 Nasdaq-100 index constituents over the period 1995–2020. To control for the potential endogeneity bias, a maximum-likelihood treatment effects model is used to simultaneously estimate the decision to form a captive and the effect of that decision on CFR in a two-equation system. The details of this method are provided in Section *3.3 Multivariate Analysis*. All regressions include year and industry dummy variables. The *p*-values are in parentheses. *, ** and *** denote statistical significance at the 10%, 5% and 1% levels, respectively.

4.2. Disaggregation Based on Firm Characteristics and Time Periods
To explore the relationships amongst cash flow levels, captive formation and certain firm characteristics, we divide the entire sample according to firm age, cash holdings, industry affiliation, and time interval. The results of various subsamples are presented in Table 6a and 6b.

Specifications (1) and (2) of Table 6a show estimates of treatment regressions based on the subsamples by firm age, while Specifications (3) and (4) present estimates based on the subsamples by cash holdings. The coefficient of the *Captive* variable is significant and positive in Specification (1) for firms with age less than the median (i.e., 18 years according to Table 3)—an outcome consistent with the hypothesis that captives help firms improve cash flow. However, for those firms with age equal or higher than the median in Specification (2), the *Captive* variable turns to have a negative sign. Although the exact reason for this is unknown, it may suggest that firms with a shorter history of operating as public firms may leverage the efficiency of captives to improve their cash flow ratios. This result also indicates that firms with less than 18 years as publicly traded entities are more likely to demonstrate the expertise of using captives to their advantage of improved cash flow. As Marsh (2017) points out, the attractiveness of captives comes from multiple benefits, not just one. Firms from different age groups may turn to captive vehicles for a variety of risk management needs.

A similar pattern of opposite signs for the *Captive* variable is also observed for the following two subsamples: firms with cash holdings less than the median in Specification (3) and firms with cash holdings equal or more than the median (i.e., 12.46 percent according to Table 3) in Specification (4). That is, a negative relationship between captive use and CFR exists for firms with lower levels of cash holdings, but a positive relationship is present for firms with higher levels of cash holdings. This finding implies that firms with more cash holdings are more likely to use captives to enhance their cash flow. The parent company with more cash appears to generate more cash flow, thanks to internalized premiums paid to its captive.

Table 6a: Captive Insurance and Cash Flow Ratio – Subsamples by Age and Cash

Variable	Subsample		Subsample	
	(1) Firms with "*Age* < Median"	(2) Firms with "*Age* ≥ Median"	(3) Firms with "*Cash* < Median "	(4) Firms with "*Cash* ≥ Median"
Panel A: CFR (Eq. 1)				
Captive	0.090***	-0.052***	-0.074***	0.101***
	(0.000)	(0.001)	(0.000)	(0.000)
Size	-0.003	0.006*	0.009***	-0.005
	(0.270)	(0.095)	(0.004)	(0.186)
Opacity	0.007	0.017	0.020	0.000
	(0.742)	(0.299)	(0.283)	(0.968)
Capex	0.334***	0.505***	0.549***	0.315**
	(0.005)	(0.000)	(0.000)	(0.023)
ROA	0.697***	0.677***	0.649***	0.715***
	(0.000)	(0.000)	(0.000)	(0.000)
Sales growth	0.004	-0.007	-0.002	0.002
	(0.512)	(0.356)	(0.707)	(0.716)
Dividend	-0.016	0.0129*	0.007	-0.008
	(0.158)	(0.080)	(0.479)	(0.343)
Leverage	-0.0398*	-0.046***	-0.0525***	-0.038
	(0.065)	(0.008)	(0.001)	(0.112)
Age	-0.000	-0.0000172	0.000	-0.000
	(0.563)	(0.954)	(0.680)	(0.204)
Panel B: Captive (Eq. 2)				
Size	0.685***	0.515***	0.501***	0.772***
	(0.000)	(0.000)	(0.000)	(0.000)
Cash	-5.012***	-6.164***	-6.456***	-0.631
	(0.003)	(0.001)	(0.003)	(0.577)
Opacity	-2.934***	-0.105	0.015	-1.622
	(0.006)	(0.897)	(0.980)	(0.127)
Capex	8.626***	0.578	3.437	6.143
	(0.000)	(0.904)	(0.296)	(0.128)
Dividend	1.366***	0.691**	0.703**	1.380***
	(0.001)	(0.049)	(0.033)	(0.000)
Leverage	-0.0648	-1.767***	-1.180***	-4.439***
	(0.929)	(0.006)	(0.004)	(0.009)
Age	-0.0028	0.000	0.001	0.013
	(0.928)	(0.988)	(0.882)	(0.154)
No. of observations	875	970	923	922
No. of clusters	80	74	86	83
Log pseudolikelihood	909	1312	1105	1056
Wald test of independent equations	31.29 ***	15.11***	75.95***	34.27***

Note: The results of treatment effects are based on 2020 Nasdaq-100 index constituents over the period 1995–2020. The *p*-values are in parentheses. *, ** and *** denote statistical significance at the 10%, 5% and 1% levels, respectively.

Table 6b demonstrates estimates for the two subsamples based on industry affiliation in Specifications (1) and (2) and for another two based on time periods of pre- and post-financial crisis in Specifications (3) and (4). According to Table 2b, three industries—GIC 10 Energy, GIC 15 Materials, and GIC 55 Utilities—have no companies with captives. As a result, we further test our hypothesis using only the industries affiliated with companies that formed captives in Specification (1). In addition, three industries—GIC 30 Consumer Staples, GIC 45 Information Technology, and GIC 50 Communication Services—account for almost three quarters of all active captives. Therefore, we estimate the model particularly for companies in these three industries in Specification (2). Consistent with our hypothesis, a positive sign exists for the *Captive* variable in Specification (2), implying that companies in these three industries can capitalize on captives to improve their CFR. Nevertheless, the coefficient of the *Captive* variable is significant but negative in Specification (1). Those results based on industry affiliation appear to coincide with what was observed in Table 6a with opposite signs for disaggregated datasets by *Age* and *Cash*. This outcome seems to imply that a captive structure is particularly suited for firms in the industries of Consumer Staples, Information Technology, and Communication Services to benefit from the efficiency of risk management via captives and boost their cash flow.

In light of the 2008 global financial crisis that may have created an economic shock on the corporate risk management landscape, we divide the entire firm-year observations into two subsamples: the pre-crisis period (1995–2008) in Specification (3) and the post-crisis period (2009–2020) in Specification (4). This division can verify whether captives can bring different effects on CFR as managerial risk attitudes toward risk may change after the global financial crisis. Once again, mixed results show up for the signs of the *Captive* variable. Those companies in the post-crisis period demonstrate higher cash flow through captives, while those in the pre-crisis do not. A feasible explanation is that companies in the post-crisis period survive the devastating financial crisis and tend to keep more cash afterwards. They can be in a better position to take advantage of captive structures to improve their cash flow. In addition, this finding appears to reinforce the results in Specification (4) of Table 6a that the use of captives has a positive effect on CFR for firms with higher cash holdings. The results based on the post-crisis period also reflect that the Nasdaq-100 has changed significantly since the financial crisis, both in terms of new constituent members as well as maturation of the constituents that have persisted over the entire study period. As indicated in the previous section on data collection, the complexion of the Nasdaq-100 index has evolved because its composition is adjusted annually according to market capitalization. This finding may further suggest that those companies with captives may have learned from the crisis and operated their captives to achieve the benefit of improved cash flow.

Table 6b: Captive Insurance and Cash Flow Ratio – Subsamples by Industry and Time Period

Variable	Subsample by industry		Subsample by time period	
	(1) Firms with GIC of 20, 25, 30, 35, 45, and 50.	(2) Firms with GIC of 30, 45, and 50.	(3) 1995–2008	(4) 2009–2020
Panel A: CFR (Eq. 1)				
Captive	-0.0744***	0.0860***	-0.097***	0.072***
	(0.000)	(0.000)	(0.000)	(0.000)
Size	0.006**	-0.011***	0.008**	-0.009***
	(0.021)	(0.001)	(0.044)	(0.003)
Opacity	-0.009	0.001	0.006	0.013
	(0.512)	(0.928)	(0.804)	(0.456)
Capex	0.474***	0.520***	0.431***	0.329**
	(0.000)	(0.000)	(0.000)	(0.010)
ROA	0.676***	0.616***	0.654***	0.713***
	(0.000)	(0.000)	(0.000)	(0.000)
Sales growth	0.002	-0.004	-0.002	0.014
	(0.607)	(0.420)	(0.514)	(0.161)
Dividend	0.005	0.004	0.006	-0.002
	(0.412)	(0.620)	(0.533)	(0.724)
Leverage	-0.040***	-0.033**	-0.051**	-0.026*
	(0.000)	(0.016)	(0.017)	(0.093)
Age	0.000	-0.000	-0.000	-0.000
	(0.762)	(0.174)	(0.730)	(0.448)
Panel B: Captive (Eq. 2)				
Size	0.525***	0.733***	0.478***	0.563***
	(0.000)	(0.000)	(0.000)	(0.000)
Cash	-6.683***	-1.685	-7.135***	-2.587
	(0.000)	(0.297)	(0.000)	(0.130)
Opacity	-0.969*	-0.944	0.462	-0.049
	(0.064)	(0.320)	(0.525)	(0.945)
Capex	2.800	1.870	1.280	10.89***
	(0.376)	(0.615)	(0.675)	(0.002)
Dividend	0.703***	0.601**	0.837**	1.004***
	(0.007)	(0.047)	(0.028)	(0.002)
Leverage	-0.548	-0.041	-1.156***	-1.599**
	(0.118)	(0.955)	(0.001)	(0.024)
Age	0.004	0.017**	-0.002	0.006
	(0.585)	(0.042)	(0.837)	(0.451)
No. of observations	1794	1131	826	1019
No. of clusters	96	60	81	98
Log pseudolikelihooc	2017	1375	862	1274
Wald test of independent equation	50.09 ***	23.80***	31.31***	28.96***

Note: The results of treatment effects are based on 2020 Nasdaq-100 index constituents over the period 1995–2020. Table 2b provides the details of the two-digit Global Industry Classification (GIC) Standard codes. Because the earliest "effective from date" of Nasdaq-100 companies on COMPUSTAT is 1/1/1995, the entire sample is broken down into two subsamples: firms with effective from date 1995-2008 and firms with effective from date 2009–2020. The *p*-values are in parentheses. *, ** and *** denote statistical significance at the 10%, 5% and 1% levels, respectively.

4.3. Robustness Tests

A survival dataset has been created to test the robustness of the results, following arguments made by Chang & Chen (2019, 2018), Berry-Stölzle and Xu (2018), and Pagach & Warr (2011). The main reason behind the use of this dataset is to focus on the decision to establish a captive among those companies during the period 1995 through 2020. The selection procedure goes through the following two steps. The first is to ensure only one firm-year observation for any firm with a captive. Therefore, firm-year observations after the year that this firm formed its captive have been removed from the dataset. The second is to exclude firms that formed captives before the year 1995. The descriptive statistics on our survival dataset are displayed in Table 7, and the estimates based on the treatment effects model show in Table 8.

The benefits of using a survival dataset comes with a cost because just about one percent of firm-year observations are represented by firms with captives, evidenced by the descriptive results in Table 7. In addition, the medians of all other variables do not differ too much from those observed in Table 3 for the entire sample, except for the median of CF that dropped a lot in the survival dataset. It is noteworthy that the survival dataset may serve the purpose of investigating the effect of a firm's decision to form a captive on cash flow, given the truncated sample. There is no doubt that the level of statistical power declines with the reduced sample size. In general, the robustness tests based on the survival dataset in Specifications (2), (3), and (4) of Table 8 do not provide evidence in support of a positive connection between captive formations and cash flow. This is consistent with what is observed for the entire sample shown in Table 5. One exception is illustrated in Specification (1) that show a positive coefficient of the *Captive* variable when *Size* is the only control variable put in the Eq. (1). These mixed results in Table 8 imply the relationship between captive formation and cash flow is not stable. Furthermore, they coincide with the results in Table 6a and 6b that the positive effects of captive use on cash flow are conditional upon the time interval, the tenure as a public firm, the volume of cash holdings, and industry affiliation.

Table 7: Descriptive Statistics on the Survival Sample

Variable	*N*	Min	Max	Mean	Median	Std. deviation
Captive	1633	0	1	0.009	0	0.095
CF	1633	-284.487	36314	1616.62	593.9	3228.1
CFR	1633	-0.329	0.393	0.12553	0.135	0.122
Size	1633	17.555	26.007	22.068	22.198	1.756
Cash	1633	0.002	0.709	0.173	0.136	0.147
Opacity	1633	0	0.773	0.184	0.100	0.208
Capex	1633	0.003	0.214	0.048	0.033	0.044
ROA	1633	-0.469	0.319	0.060	0.081	0.136
Sales growth	1633	-0.955	4.193	0.256	0.132	0.620
Dividend	1633	0	1	0.340	0	0.474
Leverage	1633	0	1.314	0.124	0.039	0.227
Age	1633	0	86	20.005	17	15.971

Note: *Captive* is a dummy variable that equals 1 for a firm-year in which a captive is used and 0 otherwise.

Table 8: Robustness Tests for Captive Insurance and Cash Flow Ratio

Variable	Survival dataset		Survival dataset	
	(1)	(2)	(3)	(4)
Panel A: CFR (Eq. 1)				
Captive	0.265***	-0.227***	-0.0789***	-0.076***
	(0.000)	(0.000)	(0.005)	(0.005)
Size	0.004	0.009	-0.001	0.000
	(0.496)	(0.124)	(0.654)	(0.760)
Opacity		-0.033	-0.005	0.000
		(0.256)	(0.672)	(0.994)
Capex		0.487**	0.400***	0.418***
		(0.006)	(0.000)	(0.000)
ROA			0.710***	0.696***
			(0.000)	(0.000)
Sales growth			0.002	0.001
			(0.636)	(0.802)
Dividend				-0.002
				(0.722)
Leverage				-0.032***
				(0.001)
Age				-0.000
				(0.343)
Panel B: Captive (Eq. 2)				
Size	0.465***	0.297**	0.350***	0.369***
	(0.000)	(0.000)	(0.000)	(0.000)
Cash	-1.773	-4.054**	-2.927*	-3.089*
	(0.304)	(0.003)	(0.093)	(0.084)
Opacity	-0.910*	-0.115	-0.529	-0.583
	(0.068)	(0.803)	(0.412)	(0.353)
Capex	4.731**	0.725	2.690	2.674
	(0.015)	(0.742)	(0.288)	(0.283)
Dividend	0.402	0.247	0.240	0.228
	(0.221)	(0.225)	(0.331)	(0.357)
Leverage	-0.917**	0.520**	-0.221	-0.386
	(0.022)	(0.021)	(0.582)	(0.318)
Age	-0.001	-0.004	-0.003	-0.003
	(0.802)	(0.361)	(0.652)	(0.536)
No. of observations	1633	1633	1633	1633
No. of clusters	99	99	99	99
Log pseudolikelihood	1165	1202	2016	2023
Wald test of independent equations	40.91***	33.53***	10.29***	10.67***

Note: The results of treatment effects are based on 2020 Nasdaq-100 index constituents over the period 1995–2020. To control for the potential endogeneity bias, a maximum-likelihood treatment effects model is used to simultaneously estimate the decision to form a captive and the effect of that decision on CFR in a two-equation system. The details of this method are provided in Section *3.3 Multivariate Analysis*. The *p*-values are in parentheses. *, ** and *** denote statistical significance at the 10%, 5% and 1% levels, respectively.

5. Conclusion

This paper tests the conjecture that a captive can be used to improve cash flow among 2020 Nasdaq-100 index constituents with their financial data over the years 1995–2020. This index is known for its composition of non-financial, large-cap, and technology-oriented companies. Viewed as a sophisticated risk retention technique, a captive is formed to insure its parent company against loss exposures. First, the parent can keep premiums under its corporate umbrella, instead of paying them out to third-party insurers. Second, the parent is insured by its captive, shielding itself from market cycles and inefficiencies related to moral hazard and adverse selection in the conventional commercial insurance marketplace. A captive aligns the financial interests of the insurer and the insured.

Our study employs a treatment effects model to explore whether companies can influence their cash flows through the use of captives, while controlling for endogeneity issues on the decision to form captives. Three key findings emerge. First, the use of captives is not prevalently accepted by those Nasdaq-100 companies, and only 15 of them adopted alternative solutions via a captive structure. Second, the assertion that firms with captives are connected with higher levels of cash flow is supported by mean comparisons in our univariate analysis, but not by the general results in the multivariate setting over the entire sample period 1995–2020. That is, firms with captives have higher levels of cash flow than their counterparts without captives in the mean difference analysis, but this outcome does not hold up when all variables are considered in the treatment effects model. Third, a positive relationship between captive formations and cash flow levels exists in some subsamples either based on the post-financial crisis period 2009–2020 or based on some firm-specific traits—firm age, cash holdings, and industry affiliation.

The results of this study may offer some references for future research on captives in the U.S. First, the vast majority of Nasdaq-100 companies still prefer conventional commercial insurance to alternative solutions via captives. Second, whether a captive helps its parent company enhance cash flow is still a subject of debate because some firm-specific factors come into play.

One cautionary note for the results of this study is that the Nasdaq-100 index is known for its composition of non-financial and technology-related stocks. This index is not regarded as a broad barometer of overall stock markets due to the selection of its constituents. However, our focus on this important index allows us to investigate how faster growing companies manage their loss exposures using an alternative risk transfer technique via captive formation.

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Consumption Preferences and Generalized Utility Functions

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Abstract

Many well-known conclusions about consumer preferences and utility representations of consumptions are developed on the assumption that possible consumptions are completely ordered. This paper looks at what could happen when such an unrealistic assumption is removed, hoping that the consequent theory will be more relevant to real life than before. Different from some of the known hypotheses and/or conclusions in the literature, this paper shows, among other results, that for an individual no matter how he prefers one consumption over another, (i) there are incomparable consumptions, (ii) his consumption preferences may not be transitive, (iii) his indifference relation of consumptions *in practice* may not be transitive. Although these results have been confirmed by different authors with varied settings, our confirmations are based purely on analytical analysis without making use of any auxiliary concepts. More importantly, this paper generalizes the classical conclusion of Debreu (1959) on when a continuous utility representation exists. In the end, several topics of expected significance are suggested for future research.

Introduction

Central to economics is the behavioral hypothesis of rationality that individual decision makers optimize their subjective functions (Sobel, 2005). However, among many existing problems with this hypothesis, such as those raised by behavioral economists (e.g., Kahneman, 2011; Mullainathan & Thaler, 2000), are the following two problems that this paper attempts to address:

- How can one alter the hypothesis that a consumer's set of consumption possibilities is completely ordered by his preferences (Dubra & Ok, 2002; Hervés- Beloso & Cruces,

2019) in order to reflect the fact that the opposite should be generally true?

- When a consumer's satisfaction from consumption is not measurable by the conventional concept of continuous utility functions (Toranzo & Beloso, 1995; Ok, 2002), what can one do to reflect the levels of satisfactions?

These are very important problems if we want to make consequent theories practically relevant. For example, Jacob et al. (2018) find that most U.S. college students value living amenities, such as spending on student activities, sports, and dormitories, over academic quality that is only a concern of the small number of high-achieving students. By looking at cosmopolitan cultural consumption – consumer's openness for cultural products from foreign countries, Rössel and Schroedter (2015) maintain that cosmopolitan consumption is a class-based practice, determined by different forms of cultural capitals. Similarly, many other researches point to the same fact that as a human being, each consumer's physiological needs are of multi-dimensional. When two consumption choices belong to two different dimensions, respectively, these choices will not be comparable in terms of consumption preferences.

Although the assumption that consumer's preference can order all available consumption choices (Hervés- Beloso & Cruces, 2019) does not reflect the relevant reality, it does play the role of starting points of countlessly many theoretical reasonings and practical applications of economic theories. Hence, to make relevant theories practically relevant, it is important both theoretically and practically to address the previously posed problems so that adopted assumptions are closer to real life than the ones widely adopted currently.

In terms of methodology, this paper employs Euclidean spaces to investigate whether or not some of the well-known conclusions of the consumer theory still hold true when consumptions are not assumed to be completely comparable in terms of individual preferences (the first problem above), and how potentially another set of indicators instead of that of real numbers can be employed to measure utilities (the second problem above). Different from some of the known hypotheses and/or conclusions in the literature, this paper shows, among other results, that for an individual, (i) there are incomparable consumptions, (ii) his consumption preferences may not be transitive, (iii) his indifference relation of consumptions *in practice* may not be transitive, and (iv) the classical conclusion of Debreu (1959) on when a continuous utility representation exists is generalized.

As for the question of why we choose the methodology of Euclidean spaces for this investigation, it can be explained from several different angles. First, as demonstrated by Forrest and Liu (2022, Chapter 1), methods, commonly employed in studies of economics and business, such as those that are language based, calculus based, and data or anecdote based, all experience inadequacies for the reason that issues in relevant studies generally deal with organizations, evolutions and interactions of organizations. That is why the methodology of systems science (Forrest, 2018; Klir, 1985; Porter, 1985) needs to be looked at. To accomplish this end, as a set-theoretical tool, Euclidean spaces emerge as the best choice for this research, because such a tool can help avoid the said inadequacies and produce generally-true conclusions, for relevant discussions, see Forrest and Liu (2022, Chapter 1). Second, the key results this work attempts to revisit are originally derived by using the language of sets and Euclidean spaces (Mas-Collel et al., 1985). Third, the structural essence of Euclidean spaces is closest to real life when compared to other commonly applied methods, such as those that are calculus-based or statistics based (Lin & OuYang, 2010), in terms of consumptions and preferences.

As for the contribution this paper makes to the literature, it can be examined in both theoretical and practical perspectives. In the former case, this work is the first in four different fronts. (1) It *analytically* shows the fact that for each consumer, there are possible consumptions that are not comparable in terms of his/her preferences. (2) It officially embraces the fact that each consumer or decision maker orders real numbers differently based on his/her system of values and beliefs. (3) Due to measurement uncertainties in real life, a constructed example shows that for an economic theory to be practically useful, the theory has to allow some of the involved variables to assume interval values instead of exact numerical ones. (4) In terms of utility representations of a preference relation, this work takes a different approach from the one taken by Efe Ok and his colleagues. In comparison, our conclusion can be more easily fathomed behaviorally than the ones derived by Ok's team (e.g., Dubra & Ok, 2002; Evren & Ok, 2011; Nishimura & Ok, 2016; Ok, 2002; Ok & Masatlioglu, 2007).

In practical perspective, because consumers are allowed to order real numbers differently, conclusions derived in this work can be practically applied to situations that involve irrational behaviors and non-optimizing consumptions (Taylor, 1989).

The rest of this paper is organized as follows. After preparing the reader for the development of logical reasoning of the rest of the paper, counterexamples and several generally true conclusions regarding incomparable consumptions and nontransitive consumer preferences are presented. And then, studied are some structural properties of consumption preferences and a generalization of the conventional concept of utilities. The last section discusses a case of real-life application that supports conclusions established in this paper.

The Basic Model Setting and the Axiom of Lower Boundedness

This section lays down the basics needed for the rest of the paper to develop smoothly. It consists of three subsections. The first one provides a quick literature review and how conclusions derived here enrich the existing knowledge. The second subsection introduces a pairwise order relation for the elements in the Euclidean space $\mathbb{R}^\ell$, where ℓ is a natural number. And the third subsection constructs an individual's set of possible consumptions and introduces the axiom of lower boundedness.

Basic Assumptions Underlying Consumers' Utilities and Preferences

In the economic literature, the word utility commonly means the satisfaction a consumer acquires from consuming a good or service. It has been in use since at least the time of Aristotle (Gordon, 1964; Kauder, 1953). Even so, its current meaning was only crystalized in the 20$^{\text{th}}$ century. With such a long history and importance in economic studies, actual measurement of utility has never been established, although various scholars have devoted time and efforts on this task throughout the history (Stigler, 1950).

To avoid the problem of not being able to observe pleasure acquired from consuming a good or service, economists turned their attention and efforts to observable aspects of decisions in terms of which commodity is selected for consumption in the name of preferences (Varian, 2010). Such shift of attention enabled economists to study concrete, observable phenomena, making economic theories, to a degree, similar to physics where the movement of bodies is addressed (Pareto, 1906).

Before the 20th century, it was assumed that one could always assign a real number to every consumption bundle available to a consumer to represent the order in which he prefers them. That is, it was believed that a preference relation could always be measured numerically. In the effort to make such belief more scientific, Fisher (1892), Pareto (1906) and others realized that in terms of utility, focusing on the ranking of consumption alternatives, instead of on how much these alternatives differ from each other, can very well address the problems to which utility theory was conventionally applied. By adopting this new approach, utility no longer needed to be numerically measurable (Strotz, 1953).

This approach of ranking consumption alternatives, known as ordinalism, has paved the way for the development of modern microeconomics. And, Slutsky (1915), Hicks and Allen (1934), Samuelson (1938) and others contributed to make this approach the dominant one in consumer theory, and opened up new theoretical possibilities through the study of preferences. In particular, a consumer's preference is assumed to completely order all available consumption alternatives and satisfies the property of transitivity (Hervés- Beloso & Cruces, 2019). To this end, this paper shows, by using an example, that in general for each consumer there are consumption alternatives that are not comparable with each other in terms of his preferences. That is, in the studies of preferences, this work is expected to open up new possibilities for the consequent economic theories to be closer to situations of real life.

Let $\precsim$ be a consumer's preference relation defined on the set X of his consumption alternatives. It is said to possess a utility representation, if there is a function $u: X \to \mathbb{R}$, where $\mathbb{R}$ represents the set of all real numbers, such that for any $x, y \in X$, $x \precsim y$, if and only if $u(x) \leq u(y)$ (Hervés- Beloso & Cruces, 2019). Here, the function u is known as a utility function. Different from the earlier times, Wold (1943) was the first to investigate the conditions under which a utility function could represent some ranking of a consumer's preferences.

Following Wold, Debreu (1959) and many others over the following years and up to the present time (Mehta, 1998) have been trying to refine and generalize the established results on when representable consumer preferences exist. Because of the continued efforts to bring economics into scientific grounds, various mathematical methods and approaches were critically introduced to hopefully reshape the discipline (e.g., Debreu, 1959; Mas-Collel et al., 1995). And with the increasing level of scientification of economics, it is confirmed (Toranzo & Beloso, 1995; Hervés- Beloso & Cruces, 2019) that there are indeed preference relations that do not have any utility representation. For a very nice historical account of this area of literature, see Hervés- Beloso and Cruces (2019).

Contributing to this branch of the literature, this paper generalizes Debreu's (1959) existence theorem of representable preference relations. Here, a particular condition is imposed based on the most recent development in mathematics.

A Pairwise Comparability of the Euclidean Space $\mathbb{R}^\ell$

To make the rest of this paper more reader friendly, let us finish this section by introducing the order relation $\leq$ on $\mathbb{R}^\ell$ as follows, where ℓ is a natural number. For any $x^1 = (x_1^1, x_2^1, \dots, x_\ell^1)$ and $x^2 = (x_1^2, x_2^2, \dots, x_\ell^2) \in \mathbb{R}^\ell$,

$$x^1 \leq x^2 \text{ if and only if } x_h^1 \leq x_h^2, \text{for each } h = 1,2, \dots, \ell.$$

Evidently, there are elements x^1 and $x^2 \in \mathbb{R}^\ell$ such that x^1 and x^2 are not comparable in terms of this order relation $\leq$.

A subset $X \subseteq \mathbb{R}^\ell$ is known as connected, if X cannot be partitioned into two nonempty open subsets in the relative topology induced on X.

Individuals' Consumptions and Consumption Sets

By a consumer, we mean an individual person, a household, or a group of people organized either purposefully or naturally together around a common purpose or reason, such as an extended family. For the sake of communication convenience, we will treat a consumer as "he".

Assume that what a consumer does is to choose and to carry out a consumption plan, or simply a consumption, selected now for the present moment and the entire future, as what has been done in the literature (Debreu, 1959; Levin & Milgrom, 2004; Mas-Collel et al., 1995). In other words, he specifies the quantities of all his input commodities and all his output commodities subject to a set of constraints, assuming that he does not have a single or a particularly preferred consumption. The constraints consist of those that are of, for example, physiological nature, such as those needed to sustain survival and basic living, and that the total value of his consumption must not be greater than his level of wealth.

Without loss of generality, assume that there are m consumers, for some $m \in \mathbb{N}$ (= the set of all-natural numbers). For consumer i (= 1, 2, …, m), to distinguish inputs (i.e., those goods and services consumed by i) and outputs (= what i offers to the world), the quantities of his commodity inputs are written as positive numbers, while the quantities of his commodity outputs negative numbers. Assume that there is a total of ℓ commodities, which are ordered and named as $h = 1, 2, \ldots, \ell$. As commonly done in economic analysis (e.g., Pancs, 2018), assume that the quantity of each commodity, as shown in a consumption plan, is a real number.

Let x_i ($\in \mathbb{R}^\ell$) represent a consumption of consumer i and X_i be the set of all consumptions possible for consumer i, known as his consumption set or his demand. Then, this set X_i is completely determined by consumer i's constraints. Based on this convention, it can be seen that each consumption $x_i \in X_i$ generally contains a relatively small number of nonzero components.

For each individual, his typical inputs of a consumption consist of various dated and location-specific goods and services, while the only outputs are various dated and located labors provided. What is assumed here is that goods, services, or labors that become available and/or are delivered at different times and/or different locations are seen as different commodities. To separate a consumer from a producer, assume that each consumer plays two roles:

- A provider of services that facilitate the transactions of purchase and sale of products, such as a house, a car, etc., and
- A consumer of services and products from others.

Let $x_i \in \mathbb{R}^\ell$ be a consumption of consumer i $(= 1,2,\ldots,m)$. Then

$$x = \sum_{i=1}^{m} x_i \text{ and } X = \sum_{i=1}^{m} X_i \tag{1}$$

are respectively known as a total consumption and the total consumption set.

When commodity h is an input for a consumer, the consumed quantity of h must have a lower bound, such as zero. On the other hand, when commodity h is an output of a consumer, then the quantity of this commodity must be bounded from above, because the individual can only produce a limited amount of output at a specified time-interval, no matter what other commodities he might input and output. For instance, if for consumer i, commodity output h is a type of labor offered to the market, then the amount of such labor has to be limited with an upper cap. Because the quantity of output commodity h is written as a negative number, it means that the quantity of h has a lower bound. Based on this understanding, we adopt the following axiom:

Axiom 1 (Lower Boundedness): For each consumer $i\ (= 1,2,\dots,m)$, his consumption set X_i has a lower bound for the order relation $\leqq$ defined on $\mathbb{R}^\ell$.

Without causing confusion, when once an axiom is introduced, this axiom will be assumed to hold true in the following reasonings unless it is stated otherwise. Hence, the result below naturally follows:

Proposition 1. The total consumption set X of all consumers has a lower bound in terms of the order relation $\leq$ defined on $\mathbb{R}^\ell$.

In fact, if $\chi_i \in \mathbb{R}^\ell$ is a lower bound of the consumption set X_i, for $i = 1,2,\dots,m$, then equation (1) implies that $\chi = \sum_{i=1}^{m} \chi_i$ is a lower bound of X. QED

Among many commonly adopted assumptions about consumption sets X_i, i = 1, 2, …, m, is that of continuity. That is, for each i = 1, 2, …, m, X_i is assumed to be closed. This end means that for any infinite sequence $\left(x_i^q\right)_{q=1}^{\infty}$ of consumptions possible for consumer i, if $x_i^q \in X_i$, $q = 1,2,\dots$, and $x_i^q \to x_i^0$, as $q \to \infty$, then $x_i^0 \in X_i$.

Evidently, in real life, this assumption of continuity cannot be true in general. For instance, assume that consumer i is a person with a very strong conceit and a comparing heart, and that $x_1^1 \in X_i$ is i's current consumption. Then driven by his natural desire to satisfy his vanity, especially when there are stimulating comparisons with others (Blanchflower et al., 2009; Esposito & Villaseñor, 2018; Schneider &

Day, 2018), i would prefer a better consumption $x_1^2 \in X_i$. As the living quality of the competing members of the community rises, comparisons with each other within the community further encourage i to pursue another preferred consumption $x_1^3 \in X_i$ above x_1^2. Over time, a sequence $\left(x_i^q\right)_{q=1}^{\infty}$ of consumptions are obtained. If such a sequence is convergent to x_i^0, most likely, $x_i^0 \notin X_i$, because such consumption x_i^0 of the limit state will be most likely not materializable, although imaginable, within the constraints of consumer i.

Incomparable Consumptions and Nontransitive Preferences

This section investigates the preference relation of a consumer's consumption. It consists of three subsections. The first subsection demonstrates by using an example that the preference of one consumption over another might not be applied to compare some consumptions. The second subsection shows also by using an example that the preference relation that naturally exists in the set X_i of consumptions in general cannot be seen as a preorder, because in real life the property of transitivity may not hold true. The third subsection shows how the indifference relation of consumptions can help partition X_i into disjoint equivalence classes, while showing that in real life, the indifference relation does not necessarily satisfy the property of transitivity.

Speaking differently, the significance of this section is the discovery that the widely adopted assumptions in the studies of consumers regarding consumer preferences (Debreu, 1959; Levin & Milgrom, 2004; Mas-Collel et al., 1995) need to be modified. Consequently, that means some of the main conclusions in such studies need to be generalized to capture additional real-life scenarios.

Existence of Incomparable Consumptions

When two consumptions $x_i^1, x_i^2 \in X_i$ are available but only one of them can be chosen, then in real life it is very likely that one is more preferred than the other. In such a case, we say that consumptions x_i^1, x_i^2 are comparable with each other in terms of consumer i's preferences.

Axiom 2 (Comparability). If two consumptions $x_i^1, x_i^2 \in X_i$ are comparable in terms of the preference of consumer i, as determined by his system of values and beliefs, then one and only one of the following alternatives holds true:

(1) x_i^1 is preferred to x_i^2;
(2) x_i^1 is indifferent to x_i^2;
(3) x_i^2 is preferred to x_i^1.

The following example shows that in real life, there is such a potential consumer that some of his consumption possibilities are simply not comparable in terms of his preferences.

Example 1: Assume that we look at a consumer who drinks coffee, although he has no particular preference of one coffee type over another. Let X be the set of consumption possibilities of this consumer, h_1 represent a coffee made from Arabic beans, and h_2 another coffee made from Robusta beans. Assume that h_1 and h_2 are respectively priced at p_{h_1} and p_{h_2}. For the convenience of presentation, we also use the same symbols h_1 and h_2 to represent the quantities of these coffees available to this consumer.

Let $x^1, x^2 \in X \subseteq \mathbb{R}^\ell$ be two such consumption possibilities that they satisfy

$$x_h^1 = x_h^2, h = 1,2, \dots, \ell, h \neq h_1, h_2,$$
$$x_{h_1}^1 = 0, x_{h_1}^2 \neq 0, x_{h_2}^1 \neq 0, x_{h_2}^2 = 0,$$

and

$$p_{h_1} x_{h_1}^2 = p_{h_2} x_{h_2}^1.$$

That is, these consumptions x^1 and x^2 are identical except their h_1 th and h_2 th components. Then, for this particular non-coffee drinker, x^1 and x^2 are not comparable consumption possibilities in terms of his preferences. It is because no matter which coffee is served, the cost is the same while he does not care or even enjoy which coffee is provided. QED

More generally, each consumer is a physiological being with multi-dimensional needs for basic survival. Hence, his set of consumption possibilities cannot be completely comparable in terms of his preferences. In other words, when two commodities from two different dimensions of human survival are presented, no consumer can really say which commodity is preferred over the other. In terms of literature, several scholars had also noticed this issue of incomplete preferences. For example, Dubra and Ok (2002) introduce a risky-choice model in which an individual naturally possesses an incomplete preference relation. Ok (2002) considers the problem of how to represent an incomplete preference relation by means of a vector-valued utility function. Based on these works, Alonso et al. (2010) present a web-based consensus support system that involves decisions makers with incomplete preferences relations; Meng and Chen (2015) develop a group-decision-making method to cope with incomplete preference information; and Cettolin and Riedl (2019) conduct experiments to test for either complete or incomplete preferences.

Nontransitive Preference Relations

For any two consumptions $x_i^1, x_i^2 \in X_i$, if they are comparable and x_i^1 is at most as preferred as x_i^2, then we write $x_i^1 \precsim_i x_i^2$. In other words, the inequality $x_i^1 \precsim_i x_i^2$ means that x_i^1 is less preferable than or indifferent from x_i^2. Symbolically, we have

$$\precsim_i \leftrightarrow (\prec_i \text{ or } \sim_i). \tag{2}$$

It can be seen that this preference relation $\precsim_i$ satisfies the following property of reflexivity: for any $x_i \in X_i$,

$$x_i \precsim_i x_i.$$

If for any $x_i^1, x_i^2, x_i^3 \in X_i$,

$$x_i^1 \precsim_i x_i^2 \text{ and } x_i^2 \precsim_i x_i^3 \text{ imply } x_i^1 \precsim_i x_i^3,$$

then $\precsim_i$ is said to be transitive. For consumer i, if his preference relation $\precsim_i$ satisfies both reflexivity and transitivity, then $\precsim_i$ is known as a preorder. If, additionally, for any consumption possibilities

$x_i^1, x_i^2 \in X_i$, one of the conditions (1) – (3) above holds true, then $\lesssim_i$ is said to be a complete preorder (relation).

In order to make our research in this paper relevant to real life, one needs to be cautioned that consumer i's preference relation $\lesssim_i$ might not be transitive in some cases of practical applications. To demonstrate this end, let us look at such a consumer whose particular system of values and beliefs makes the outputs of the objective function not ordered as how real numbers are ordered ordinarily. In particular, let $a \in \mathbb{R}$ be a positive real number. We define a preorder $<_{mod(a)}$ on $\mathbb{R}$ as follows (Forrest, Hafezalkotob et al., 2021): For any x and $y \in \mathbb{R}$,

$$x <_{mod(a)} y \text{ if and only if } x\, mod(a) < y\, mod(a), \tag{3}$$

where the order relation $<$ is the conventional one defined on $\mathbb{R}$, $x\, mod(a)$ is equal to the remainder of $x \div a$ and $y\, mod(a)$ the remainder of $y \div a$. When all the involved numbers a, x and y are integers, this order relation $<_{mod(a)}$ degenerates into the one widely studied in number theory (Burton, 2017).

Real-life applications of modular operations include 12-hour clocks, 7-day weeks, months of various numbers of days, and durations, which are often of variable time lengths, of projects that follow one after another. In each of these cases, when a cycle is fully traversed, a new round of counting or measurement begins again from the starting mark 0.

As for how a consumer's particular system of values and beliefs can order real numbers differently from the ordinary one, let us look at the comparison between \$30,000 and \$3 million. In particular, the ordinary ordering of real numbers tells \$30,000 < \$3 million. However, if it is known that the former figure is the wage of a lawful job, while the latter is the reward from robbing a bank, then people with certain systems of values and beliefs will automatically have \$30,000 > \$3 million. That is, these people order real numbers differently from the ordinary way.

Example 2. Let r be a fixed positive real number and the set $\mathbb{R}$ of all real numbers be partitioned into the following equivalence classes (Kuratowski & Mostowski, 1976) by using the modular operation

$mod(r)$. Let us still use $s \in [0, r)$ to represent the equivalence class below:

$$s = \{x \in \mathbb{R}: s = \text{remainder of } x \div r\} = \{x \in \mathbb{R}: x \; mod(r) = s\}.$$

In other words, these equivalence classes, when represented by numbers in the interval $[0, r)$, can be arranged in a circle with r being the circumference length, Figure 1, where the arrows stand for the direction of order from small magnitudes to large ones.

For a consumption $x_i \in X_i$, let h (= 1, 2, …, ℓ) be such a particular commodity that the demanded quantity of h satisfies the following preference relation: for two demanded quantities x_{ih}^1 and x_{ih}^2 of commodity h, $x_{ih}^1 \prec_{mod(r)} x_{ih}^2$ if and only if on the shorter arc between $x_{ih}^1 \; mod(r)$ and $x_{ih}^2 \; mod(r)$, the arrow points from $x_{ih}^1 \; mod(r)$ to $x_{ih}^2 \; mod(r)$. In real life, this particular commodity could be a specific kind of carbohydrate consumer i can physically intake in a day and r the maximum amount of this food required for a day. So, for any real number $x > 0$, $x \; mod(r)$ stands for the amount left over after the consumption of the food within a number of days.

Hence, the given points $x_{ih}^1, x_{ih}^2, x_{ih}^3$ on the circle in Figure 1 satisfy

$$x_{ih}^1 \prec_{mod(r)} x_{ih}^2 \; and \; x_{ih}^2 \prec_{mod(r)} x_{ih}^3.$$

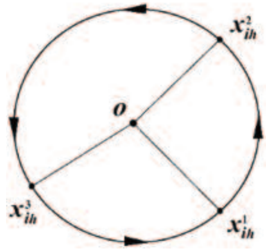

Figure 1. How the real-number line looks like after applying a modular operation

Define three consumptions $x_i^a, x_i^b, x_i^c \in X_i$ such that

$$x_{ik}^j = x_{ik}, j = a, b, c, k = 1, \dots, h-1, h+1, \dots, \ell,$$

and

$$x_{ih}^{a} = x_{ih}^{1}, x_{ih}^{b} = x_{ih}^{2}, x_{ih}^{c} = x_{ih}^{3}. \quad (4)$$

That is, the consumptions x_i^a, x_i^b and x_i^c are identical except their components of commodity h, which satisfy equation (4). Then we can conclude that

$$x_i^a \prec_i x_i^b \text{ and } x_i^b \prec_i x_i^c.$$

But, $x_i^c \prec_{mod} x_i^a$ holds true instead of $x_i^a \prec_{mod} x_i^c$ as expected as the consequence of transitivity.

One real-life example of such a commodity h that possesses a non-transitive preference relation often appears in the fashion industry, such as that of women's clothing. In particular, what was in fashion a while ago might be in fashion again many years later. QED

In terms of the literature, Tversky (1969) reports that consumer preferences don't generally satisfy the condition of transitivity. And, by using a new statistical technique and by revisiting the same gambles Tversky studied earlier, Birnbaum and Gutierrez (2007) conclude that there are indeed a few individual consumers who repeat intransitive preference patterns. More recently, the rationality assumption means (Mandler, 2001) that consumers can rank any pair of possible consumptions and the rankings satisfy the property of transitivity. Hence, either Example 1 or Example 2 or both of them demonstrate that the widely assumed rationality in economic theories does not hold true in real life.

By combining Examples 1 and 2, the result below follows naturally:

Proposition 2. For consumer i, his/her preference relation $\precsim_i$ cannot compare every pair of possible consumptions. And, although $\precsim_i$ is reflexive, it is not generally transitive.

Speaking differently, $\precsim_i$ might be neither a complete preorder nor a preorder.

Nontransitive Indifference Relations

For two consumptions x_i^1 and x_i^2 of consumer i, if $x_i^1 \gtrsim_i x_i^2$ and not $x_i^2 \gtrsim_i x_i^1$, then x_i^1 is said to be preferred to x_i^2, denoted by $x_i^1 \succ_i x_i^2$. If $x_i^1 \lesssim_i x_i^2$ and $x_i^2 \lesssim_i x_i^1$, then x_i^1 is said to be indifferent of x_i^2, denoted by $x_i^1 \sim_i x_i^2$. The relation $\sim_i$, defined on X_i, will be referred to as the indifference relation of consumer i. Notice that this indifference relation $\sim_i$ is only defined for comparable consumptions. When two consumptions x_i^1 and x_i^2 are not comparable, they are evidently also indifferent because none of them is preferred over the other. For our purpose in this paper, incomparable consumptions x_i^1 and x_i^2 will not be seen as indifferent of each other.

For any $x_i \in X_i$, define the indifference class of x_i as follows:

$$[x_i] = \{x_i' \in X_i : x_i' \lesssim_i x_i \text{ and } x_i \lesssim_i x_i'\}.$$

That is, the indifference class $[x_i]$ contains only those consumptions that are comparable with x_i and indifferent from x_i in terms of the preference relation $\lesssim_i$.

Proposition 3. For consumer i, assume that his/her preference relation $\lesssim_i$ constitutes a preorder. Then, for any consumptions $x_i^1, x_i^2 \in X_i$, if $[x_i^1] \neq [x_i^2]$, then $[x_i^1] \cap [x_i^2] = \emptyset$.

Proof. By contradiction, assume that there are $x_i^1, x_i^2 \in X_i$ such that $[x_i^1] \neq [x_i^2]$ and $[x_i^1] \cap [x_i^2] \neq \emptyset$. Then, each $z_i^* \in [x_i^1] \cap [x_i^2]$ satisfies $x_i^1 \sim_i z_i^* \sim_i x_i^2$. That is, we have

$$x_i^1 \lesssim_i z_i^*, x_i^1 \gtrsim_i z_i^*, \text{and } z_i^* \lesssim_i x_i^2, z_i^* \gtrsim_i x_i^2.$$

Therefore, the assumed transitivity implies that $x_i^1 \lesssim_i x_i^2$ and $x_i^1 \gtrsim_i x_i^2$ or $x_i^1 \sim_i x_i^2$. That is, the indifference relation $\sim_i$ is transitive, which implies that for any $z_i \in [x_i^1]$, $z_i \sim_i x_i^1 \sim_i x_i^2$. Hence, $z_i \in [x_i^2]$. That is, $[x_i^1] \subseteq [x_i^2]$. Similarly, we can show that $[x_i^1] \supseteq [x_i^2]$. Therefore, we can conclude $[x_i^1] = [x_i^2]$. A contradiction. That means $[x_i^1] \cap [x_i^2] = \emptyset$. QED

The following example shows that in general the indifference relation, as defined by incomparability, of an individual consumer is not transitive. For convenience, this indifference relation is also denoted by $\sim_i$ but in this example only.

Example 3: Continue using the setup in Figure 1, as produced by employing the modular operation $mod(r)$ on $\mathbb{R}$ in Example 2. Without loss of generality, let us identify consumptions $x_i^1, x_i^2 \in X_i$ that satisfy

$$x_{ik}^1 = x_{ik}^2, k = 1, \dots, h-1, h+1, \dots, \ell,$$

and

$$x_{ih}^1 \neq x_{ih}^2,$$

with points x_{ih}^1 and x_{ih}^2 on the circle. To this end, instead of the locations of x_{ih}^1 and x_{ih}^2 in Figure 1, let these points are given in Figure 2. That is, in this case, x_{ih}^1 and x_{ih}^2 (or x_i^1 and x_i^2) are located on the opposite sides of a diameter of the circle. Therefore, consumptions x_i^1 and x_i^2 are incomparable and so indifferent consumptions for consumer i.

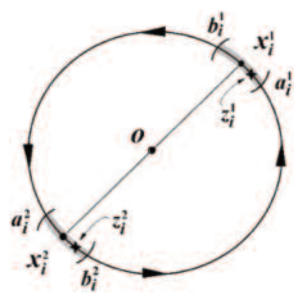

Figure 2. Arc intervals of indifferent quantities of demands

Due to measurement uncertainty and other factors in real life, for more detailed discussions, please consult with the concepts of grey numbers and systems in Liu and Lin (2010), the quantity of the demanded commodity h can never be provided in any exact amount. For example, a box of breakfast cereal is planned to contain 14 ounces of contents. However, in real life, hardly any such cereal box truly contains this exact amount as specified. Instead, the exact amount of the contents in a box of this special cereal is equal to a number very close to 14 ounces.

So, there are arc intervals, one of which is centered around x_i^1 and the other around x_i^2. Assume that the former arc interval is (a_i^1, b_i^1) and the latter is (a_i^2, b_i^2), Figure 2. Symbolically, for consumer i, for any $z_i^1 \in arc(a_i^1 b_i^1)$ and any $z_i^2 \in arc(a_i^2 b_i^2)$, we have

$$z_i^1 \sim_i x_i^1 \; and \; z_i^2 \sim_i x_i^2. \tag{5}$$

However, if $z_i^1 \in arc(a_i^1 x_i^1)$ and $z_i^2 \in arc(x_i^2 b_i^2)$, then z_i^1 and z_i^2 are not indifferent. In fact, in this case, we have

$$z_i^1 \succ_i z_i^2. \tag{6}$$

Hence, equations (5) and (6) jointly imply that

$$z_i^1 \sim_i x_i^1, x_i^1 \sim_i x_i^2, and \; x_i^2 \sim_i z_i^2 \nrightarrow z_i^1 \sim_i z_i^2.$$

That is, the indifference relation $\sim_i$ is not transitive. QED

In terms of the literature, the topics of nontransitive indifferences have been noticed and investigated by various authors. For example, such intransitivity can arise from perception difficulties, as noted by Luce (1956), or procedural decision making, when similarities are compared or regrets are considered (e.g., Loomes & Sugden, 1982; Rubinstein, 1988), or time inconsistencies caused by relative time discounting (e.g., Ok & Masatlioglu, 2007; Roefofsma & Reed, 2000). More importantly, this example shows the necessity for economic theories to consider the potential for involved variables to take interval values or to consider measurement inaccuracies. Otherwise, the practical usefulness of the theories will be limited, while producing erroneous guidelines and estimates.

One consumption $x_i \in X_i$ is said to be a satiation consumption of consumer i if there is not any other $y_i \in X_i$ such that consumer i prefers y_i to x_i. Evidently, if consumer i has incomparable consumptions, then he may very well have several incomparable satiation consumptions simultaneously.

In this research, preferences considered do not take the resale value of commodities into account. Each consumer is only interested in their personal use values. Because our convention on commodities are specific in terms of date or location or both, consumers' interests in certain commodities are date- and/or location specific. Considering the seemingly never-ending human desires for better living, let us adopt the following axiom from (Debreu, 1959).

Axiom 3 (Insatiability of preferences). For any chosen consumer, he does not have any satiation consumption.

Speaking differently, this axiom means that no matter what consumption $x_i \in X_i$ is concerned with, there is another consumption $y_i \in X_i$ such that $x_i \prec_i y_i$. That is, consumer i prefers y_i to x_i.

Structural Properties of Preferences

As the title suggests, this section studies the structure of the preference set X_i. Specifically, this section consists of two subsections, the first of which looks at how the preference relation of individual consumptions can be elevated to the level of indifference classes. The second subsection generalizes the conventional concept of utility functions with real-number ranges to that of more general ranges of indicative elements.

Preference Partitions of Consumption Sets

This subsection studies the following question that when the consumption set X_i is partitioned into equivalence classes by the indifference relation $\sim_i$, how the preference relation $\precsim_i$ can be employed to order these equivalence classes of X_i.

Proposition 4. Assume the same as in Proposition 3. Then i's set X_i of consumption possibilities can be partitioned into indifference classes such that for any $x_i^1, x_i^2 \in X_i$, if $x_i^1 \prec_i x_i^2$, then that any $z_i^1 \in [x_i^1]$ and any $z_i^2 \in [x_i^2]$, $z_i^1 \prec_i z_i^2$.

Proof. The possibility to partition X_i into equivalence classes follows from Proposition 3. The condition that $x_i^1 \prec_i x_i^2$ implies that $[x_i^1] \neq [x_i^2]$. Let $z_i^1 \in [x_i^1]$ and $z_i^2 \in [x_i^2]$ be arbitrary. Hence, the definition of the indifference relation $\sim_i$ implies that

$$z_i^1 \precsim_i x_i^1 \text{ and } x_i^2 \precsim_i z_i^2.$$

So, the transitivity of $\precsim_i$ and the condition that $x_i^1 \prec_i x_i^2$ jointly imply that $z_i^1 \prec_i z_i^2$. QED

Proposition 5. Assume the same as in Proposition 3. Then, if x_i^1 and $x_i^2 \in X_i$ are not comparable in terms of $\precsim_i$, then $[x_i^1] \cap [x_i^2] = \emptyset$; and for any $z_i^1 \in [x_i^1]$, $z_i^2 \in [x_i^2]$, z_i^1 and z_i^2 are also not comparable.

Proof. We show $[x_i^1] \cap [x_i^2] = \emptyset$ by contradiction. Assume that there is at least one $z_i \in [x_i^1] \cap [x_i^2]$. Then $x_i^1 \sim_i z_i \sim_i x_i^2$. So, the transitivity of $\precsim_i$ implies that x_i^1 and x_i^2 are comparable. A contradiction. Hence, $[x_i^1] \cap [x_i^2] = \emptyset$.

For the second conclusion, we also argue for it by contradiction. Without loss of generality, assume that there are $z_i^1 \in [x_i^1]$ and $z_i^2 \in [x_i^2]$ such that $z_i^1 \precsim_i z_i^2$. Then the definition of the indifference relation $\sim_i$ implies that

$$x_i^1 \precsim_i z_i^1 \text{ and } z_i^2 \precsim_i x_i^2.$$

These inequalities, the assumption that $z_i^1 \precsim_i z_i^2$ and the transitivity of $\precsim_i$ jointly imply that $x_i^1 \precsim_i x_i^2$. This end contradicts the assumption that x_i^1 and x_i^2 are not comparable. Therefore, for any $z_i^1 \in [x_i^1]$ and $z_i^2 \in [x_i^2]$, z_i^1 and z_i^2 are also not comparable. QED

Propositions 3 and 4 collectively indicate that there is a subset $X_i^* \subseteq X_i$ such that for any x_i^1 and $x_i^2 \in X_i^*$, $x_i^1 \neq x_i^2$ implies that $[x_i^1] \neq [x_i^2]$ and $X_i = \cup_{x_i \in X_i^*}[x_i]$. This subset X_i^* is referred to as a set of (consumer i's) preference representations. Evidently, in general the existence of X_i^* is not unique. As a corollary of Propositions 3 and 4, we have

Proposition 6. If X_i^* ($\subseteq X_i$) is a set of i's preference representations, then $\precsim_i$ is a complete preorder on X_i, if and only if $\precsim_i$ is a complete preorder on X_i^*.

The Concept of Utility – A Generalization

Define a function $u_i: X_i \to X_i^*$ as follows: for any consumption $x_i \in X_i$,

$$u_i(x_i) = x_i^* \in X_i^*, \text{if } x_i \in [x_i^*].$$

We treat u_i as a utility function of consumer i; and Propositions 3 and 4 jointly imply that for any $x_i^1, x_i^2 \in X_i$, $x_i^1 \lesssim_i x_i^2$ if and only if $u_i(x_i^1) \lesssim_i u_i(x_i^2)$.

Because the existence of the set X_i^* of preference representations is not unique, in real-life applications, one can readily employ a convenient set of preferred commodities from different areas of life as basic marks of measurement for preferences. In other words, in terms of practical applications, a certain more practically indicative set U_i that is order-isomorphic to X_i^* can be used in the place of X_i^*, where U_i does not have to involve any real numbers at all. That is, this method of using such a U_i to evaluate whether or not a particular consumption is preferred is more natural than that of using the conventional real-number valued functions of utilities.

On the other hand, the previous paragraph indicates that although the choice of X_i^* is generally not unique, the utility function u_i exists uniquely up to an order isomorphism. For example, if consumer i's preference $\lesssim_i$ is a complete preorder, then one can easily use a subset of $\mathbb{R}$ to be the range of u_i such that u_i is an increasing function. That is, in this case, X_i^* can be replaced by a set of some real numbers; and u_i is seen as an increasing function from completely preordered X_i into the set $\mathbb{R}$ of real numbers.

Let $X_i' \subseteq X_i^*$ be a subset satisfying that any two consumptions x_i^1, $x_i^2 \in X_i'$ are comparable in terms of the preference relation $\lesssim_i$. Then X_i' is known as a chain in X_i^*. A subset X_i' of X_i^* is referred to as a maximal chain, provided that for any $x_i \in X_i$, if x_i is comparable with each element in X_i', then $x_i \in X_i'$. For more details on ordered sets, please consult with Kuratowski and Mostowski (1976).

For a chosen maximal chain X_i^{max} in X_i^*, the u_i-preimage of the chain X_i^{max} is equal to

$$u_i^{-1}(X_i^{max}) = \cup \{[x_i^*]: x_i^* \in X_i^{max}\}.$$

Assume that for each $x_i' \in u_i^{-1}(X_i^{max})$, the following sets are closed in $u_i^{-1}(X_i^{max})$:

$$\{x_i \in u_i^{-1}(X_i^{max}): x_i \precsim_i x_i'\} \text{ and } \{x_i \in u_i^{-1}(X_i^{max}): x_i \succsim_i x_i'\}. \quad (7)$$

Then the well-known and classic conclusion (Debreu, 1959) on when a continuous rea-number valued utility function exists can be generalized as follows.

Proposition 7. If the following hold true, then there is a continuous utility function $u_i^*: u_i^{-1}(X_i^{max}) \to [a, b] \subset \mathbb{R}$, where a, b are two arbitrary real numbers such that $a < b$.

- Each infinity can be actually (not potentially) achieved;
- Subset $u_i^{-1}(X_i^{max})$ is connected in $\mathbb{R}^\ell$;
- For each $x_i' \in u_i^{-1}(X_i^{max})$, the sets, as defined in equation (7), are closed in $u_i^{-1}(X_i^{max})$.

Proof. In Debreu (1959, p. 56-59), the set X_i of consumer i's consumptions is assumed to be completely preordered with the preference relation $\precsim_i$. Hence, by identifying $u_i^{-1}(X_i^{max})$ with the set X_i in Debreu (1959, p. 56-59), the original argument for the existence of the desired utility function u_i^* will go through in its entirety, except that both steps 1 and 2 (Debreu, 1959, p. 57-58) cannot be successfully completed without the assumption that each infinity can be actually (not potentially) achieved.

In particular, Debreu's argument consists of 4 parts:

(a) There is a countable and dense subset D in $u_i^{-1}(X_i^{max})$, where the case that X_i^{max} is a singleton is ignored, and each point $x \in D \subset \mathbb{R}^\ell$ contains only rational-number components;
(b) An increasing function $u_i': D \to [a, b]$ is defined, for any chosen real numbers a and b such that $a < b$;
(c) This function $u_i': D \to [a, b]$ is extended to $u_i^*: u_i^{-1}(X_i^{max}) \to [a, b]$;
(d) Shown is that u_i^* is continuous.

For our purpose, let us focus on the first two steps. According to the set theory accepted as true until 2008, Debreu's original argument is perfectly fine. However, according to Lin (2008), potential infinities and actual infinities are fundamental different concepts; and they can lead to and have indeed led to completely inconsistent outcomes (Forrest, 2013), while both the existence of D in Step (a) and the construction of $u_i': D \to [a, b]$ in Step (b) mistakenly treated potential infinities as actual ones.

To understand this statement, we first look at the concept of infinities. It deals with two kinds of infinities – actual infinities and potential infinities (Lin, 2008). Specifically, each potential infinity means a present progressive tense or a forever, ongoing and never-ending process; and every actual infinity represents a present or past perfect tense or a process that actually ends or had ended.

In Step (a) of Debreu's original proof, the underlying argument for the countability of D is based on that one can match *every* rational number with a unique natural number (Kuratowski & Mostowski, 1976). Such process of matching stands for a present progressive tense, which is forever ongoing – a potential infinity. However, to derive the needed conclusion – the set of rational numbers is countable, this forever ongoing – a potential infinity – is forced to end so that what is imposed is the potential infinity = an actual infinity. According to Lin (2008) and Forrest (2013), this is a mistake and can lead to inconsistent conclusions.

In Step (b) of Debreu's original proof, if D contains either a least element x^α and/or a greatest element x^β, define $u_i'(x^\alpha) = a$ and $u_i'(x^\beta) = b$. (Note: The assumption that consumer i does not have any satiation consumption means that such a greatest element x^β cannot exist.) Next, order the other elements of D and all rational numbers in (a, b) as follows, since both sets are countable:

$$(x^1, x^2, \dots, x^p, \dots) \text{ and } (r^1, r^2, \dots, r^q, \dots). \tag{8}$$

Then, in an orderly fashion, define $u_i'(x^p)$, for each $p = 1,2,3, \dots$, so that for every r^q, $q = 1,2,3, \dots$, there is a x^p so that $u_i'(x^p) = r^q$. That is, the function u_i' is constructed in a forever ongoing process - a potential infinity. So, the eventual existence of this function u_i' can only be guaranteed under the assumption that this particular potential infinity is equal to an actual infinity.

In short, for Steps (a) and (b) of Debreu's original proof to hold true, we must assume that each infinity can be actually achieved. QED

All established propositions above lead to the chain structure of the consumption set X_i, as shown in Figure 3. In particular, for two consumptions x_i^1 and $x_i^2 \in X_i$, satisfying $x_i^1 \precsim_i x_i^2$, there might be several chains that connect into x_i^1, going from x_i^1 to x_i^2, and leaving x_i^2. Even so, when each maximal chain is concerned with, Proposition 7 says that there is a continuous and increasing function from this chain into the set $\mathbb{R}$ of all real numbers.

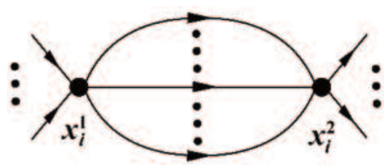

Figure 3. The chain structure of consumption set

In terms of the literature, Eilenberg (1941) considered cases for a continuous strict total order in connected and separable spaces. Wold (1943) listed a number of conditions under which a preference order possesses a real-number valued utility representation although he did not explicitly assume continuity. And, Debreu (1959) represents such a piece of work that has been seen as classical (Hervés- Beloso & Cruces, 2019). By using the same terms and symbolism as in Proposition 7, Monteiro's (1987) and Candeal et al.'s (1998) theorems for the existence of a continuous utility representation of a preference order can be accordingly generalized. All relevant details are omitted because in spirit, they are similar to those given in the proof of Proposition 7. On the other hand, in terms of utility representations of an incomplete preference relation, Proposition 7 represents a totally different conclusion than the ones in Ok (2002).

A Case of Real-Life Application

This section, which is mainly based on (Forrest, Ashimov et al., 2021), analyzes the great success of a real-life business case, where some of the theoretical conclusions established earlier provide the underlying principles that help guide the realization of the said success. To this end, one needs to note that the involved business people and firm may or may not know our established conclusions specifically. Even so, through their years of practice, they have surely sensed existence and real-life power of these results.

The Market Call that is to be Answered

With the rapid development of communication technology, the phenomenon of big data has been spreading across the entire spectrum of the business world. The company of our concern, a textbook publisher, received the following market invitation – the world of undergraduate business education urgently needs textbooks on how to deal with big data. To answer the invitation, the company first construed creatively the essence of the invitation; second, it developed its original reply to the market call, and then produced the imagined product. And, as the last step, the company effectively offered the product to the marketplace to satisfy the creatively comprehended market demand.

Evidently, the market segment the company aimed at for the purpose of maximizing its profit consists mostly of business students, reached through their instructors. The identification of this market segment actualized the conclusion of Example 1 in real life. In particular, a mindless burger flipper working in a local fast food restaurant, for example, would not have any preference of one choice of business-analytics textbook over another. On the other hand, for any instructor of relevant contents, the collection of all possible choices of business-analytics textbooks is neither complete nor transitive in terms of the instructor's preferences, confirming the conclusion of Example 3. At the same time, the instructor's utility from the usage of his/her adopted textbook satisfies the setup of Proposition 7. To illustrate the scenario more intuitively, let us imagine the availability of three different business-analytics textbooks, entitled B1, B2 and B3, respectively. Assume that these books have the following individually different characteristics:

- B1 introduces concepts logically and systematically, while applying concepts rigorously.
- B2 introduces concepts only with roughly phrased explanations, while applying concepts loosely to different scenarios.

- B3 introduces concepts loosely with vividly presented supporting scenarios, while applying concepts intuitively to various well-constructed scenarios.

As a quick reference, these textbooks and their characteristics are shown in Table 1.

Table 1: Characteristics of Textbooks B1, B2 and B3

	B1	B2	B3
Intro of concepts	Logical & rigorous	Explanatory w/out rigor	Illustrated vividly
Use of concepts	Rigorously	Loosely	Loosely

An instructor with a strong science inclination would likely order these textbooks as follows: B1 $\gtrsim$ B2 $\gtrsim$ B3, because of the degree of rigor involved in the introduction and application of concepts. On the other hand, in terms of the quality of students' learning, the instructor would realistically order the textbooks differently: B3 $\gtrsim$ B2 $\gtrsim$ B1. In particular, when students don't have the adequate maturity in rigor and logical thinking, vividly illustrated concepts, although not introduced logically and systematically, can be more readily understood by students. The point made here can be well supported if one compares the textbooks of arithmetic used in elementary schools and those used in college for mathematics majors.

As discussed here, the real challenge the publisher of our concern faced is how to entertain different standards and needs of individual instructors and students. To help protect the underlying trade secret, if there is any, the name of the company and related details of operation are omitted. For the convenience of communication, this particular company is referred to as Company Z.

Development and Marketing of a Successful Textbook

To develop a successful textbook in business analytics, Company Z drafted a potentially effective customer value proposition (CVP). It found enthusiastic authors to propose new textbooks on business analytics and instructors for information on what topics should be considered important for such a course. It put in its due diligence through investigating

1. How the ideal textbook should look like in this emerging area of learning,
2. What challenges from the side of instructors and those of students need to be met, and
3. What the best practices there are in related areas of teaching and learning.

The potential authors who addressed most of the comments from those involved in the study of the previous three questions were invited to enrich their proposals by considering suggestions missing in their initial proposals. Doing so led to a few acceptable proposals targeting different teaching methodologies and learning styles. Because each textbook had to meet multi-faceted demands of instructors and students, each accepted proposal was further developed in order to produce an "ideal" final product. Consequent to all market-sensing activities, Company Z eventually settled down and produced at least one textbook on the particular subject with necessary supporting auxiliaries. For the convenience of communication, assume that Company Z produced only one such textbook.

As the next step of operation, Company Z was to offer its textbook to as many potential instructors as possible for their adoptions.

Because the production of Company Z's textbook was entirely based on such a CVP that was co-created by the publisher and potential customers – instructors and students, most of customer concerns, if not all, had been taken care of appropriately. Offering the produced textbook to the entire market of higher education was to simply attract and convince as many instructors to adopt the textbook as possible. To accomplish this end, groups of instructors who were currently teaching or would be potentially teaching business analytics were first identified by Company Z's sales representatives. These instructors were then invited to participate in open, interactive discussions on the issues below:

1. Goals of teaching in classrooms, challenges instructors face, and what might help with achieving the goals and with overcoming the challenges in today's environment of learning.
2. Highlights of the focal textbook and how the textbook will assist instructors to accomplish their goals of teaching and students to achieve academic successes in their learning.

3. What challenges students in such a course face, and what experiences instructors have can be used to improve the solution already designed to handle the challenges.
4. Assessments and what better tools are available.
5. The increasing market demand for accessibility, mobility, and personalization.
6. Available technologies and how they help improve instruction and engage students.
7. Academic integrity and test proctoring.
8. What can be done to materialize ideals in terms of the instruction and learning of business analytics. In answering this question, each discussant thinks as a budding entrepreneur.

Five representatives, an event planner, a marketer, a product developer, a technician, and a supervisor, from Company Z, participated in activities of this marketing effort along with the invited instructors. For discussions to take place smoothly, the event planner dealt with all the detailed logistics, including travels, lodging, foods, conference facilities, entertainments, etc. The marketer headed all sessions of interactive discussions and harmonized group activities. The product developer familiarized the discussants with the available products and technology and picked up ideas about what could be either improved or introduced. The technician confirmed readily accessible technical supports and what additional supports instructors might be looking for. Finally, the supervisor holistically directed the progression of the afore-described marketing activities and looked for ways to improve the effectiveness of the event

Here is a one-line summary of the event's outcome: Most participating instructors felt that the very textbook presented to them by Company Z was the ideal book for them to use in their classroom instructions, because their concerns and issues, if not all, with regard to such a textbook had been adequately addressed.

In terms of which conclusions derived earlier in this paper have been employed either consciously or unconsciously in this business practice, here are the correspondence: The identification of market $\leftrightarrow$ Example 1; content development $\leftrightarrow$ Proposition 1; instructors' choice of textbooks $\leftrightarrow$ Examples 2 and 3 and Proposition 2; the number of textbooks of different focuses to publish $\leftrightarrow$ Propositions 4 and 5; and instructors' satisfaction $\leftrightarrow$ Propositions 6 and 7.

Conclusion

By employing the methodology of Euclidean spaces, this paper makes important strides towards answering the questions posed in the introduction section earlier. First, several counterexamples on the analytical basis are constructed to demonstrate that

- In real life, there is such a potential consumer that some of his consumption possibilities are simply not comparable with each other in terms of his preferences.
- In real life, a consumer's preferences are not generally transitive.
- Generally, a consumer's indifferent preferences in reality, as defined by incomparability, are not transitive.

Although these results have been confirmed by different authors in varied perspectives (for related references, see relevant discussions above), our confirmations are purely based on analytical analysis without making use of any auxiliary concepts; hence, the confirmations constructed in this paper are more reliable that those developed before. For example, when data are collected to show one of the results listed above, some unidentified and unidentifiable conditions regarding the environment from which the data are from, various unknown issues related to how the data are collected, and certain limitations about how the data are organized and analyzed have to be collectively involved in the production of the consequent results. Therefore, the derived conclusions carry the joint effects of all these and other unnamed constraints. In comparison, purely analytical analysis does not suffer from these constraints so that the derived conclusions are more likely to be generally true. In general, as is well-known in the research of paradoxes (Forrest, 2013, Chapter 11), when seemingly minor concepts are involved, unexpected outcomes can be produced. In other words, the literature includes various supports on how the afore-listed results could be true in different settings; however, although each of these specifically selected settings could potentially lead to desired conclusions, these conclusions will only be of limited validity. In comparison, our counterexamples do not suffer from such potential issues; so, our observations are generally more reliable than those observed within any specified setting.

Second, if a consumer can compare any two possible consumptions by his preferences, then his consumption preferences can be represented by a real-number valued utility function (Debreu, 1959). However, this big “if” just does not hold true in real life so that the ensuing “then” no longer follows naturally, as argued throughout this paper. To answer this challenge, this paper shows that there is indeed a different way to represent the preferences of a consumer and this new representation can be employed in places of the conventional utility functions. Additionally, this new representation is unique up to an order isomorphism. The importance of this discovery is that when authors forced themselves to uncover real-number valued utility representations for consumption preferences that are incomplete and nontransitive (e.g., Nishimura & Ok, 2016), the produced outcomes are no longer intuitively clear and more difficult for even theorists to understand behaviorally. That is, such efforts, at least for the present time, loose their practical significance.

Third, as shown in the literature, a consumer’s satisfaction from consuming a good or service is not generally measurable by the conventional concept of continuous utility functions (Toranzo & Beloso, 1995). However, each maximal chain of transitively comparable consumptions still enjoys a conventional utility representation, as shown in this paper. Hence, on each such chain of consumption possibilities, the present consumer theory still applies.

What needs to be emphasized here is our focus on developing an economic theory that is practically applicable in terms of producing tangible results that increase a firm’s economic value and performance through, for example, the development of innovative products or services and better predictions of consumer behaviors, instead of another repeat of the history of taking beauty for truth. This end is exactly what Paul Krugman commented in *New York Times* (2009-09-02), as outlined below. The importance of our effort to develop a practically useful economic theory has been well illustrated by major economic disasters of the recent past, such as, among many others, the unnecessary economic sufferings and property/life losses experienced by many nations that attempted, but failed miserably, to kick start their longed industrial revolutions based on incorrect theories (Forrest, Zhao et al., 2018; Wen, 2016). In other words, when beauty is mistaken as truth, a lot of disastrous consequences will unavoidably occur. To this end, see Lin and Ouyang (2010) for more in-depth discussions.

> "The economic profession went astray because economists, as a group, mistook beauty, clad in impressive-looking mathematics, for truth … As memories of the Depression faded, economists fell back in love with the old, idealized vision of an economy in which rational individuals interact in perfect markets … Unfortunately, this romanticized and sanitized vision of the economy led most economists to ignore … things that can go wrong. They turned a blind eye to the limitations of human rationality that often leads to bubbles and burst; to the problem of institutions that run amok; to the imperfection of markets … that can cause the economy … to undergo sudden, unpredictable crashes; and to the dangers created when regulators don't believe in regulation."

As for potential future research, one can readily compare what are derived in this paper with the known existence and non-existence theorems of utilities representations (Candeal et al., 1998; Eilenber, 1941; Toranzo & Beloso, 1995; Monteiro, 1987; Herves-Beloso & Monteiro, 2010). In particular, these known theorems are derived on the assumption that preferences of consumptions are complete. So, that opens up the opportunity for economists to consider how these theorems would look like when the preferences of consumptions of concern are not complete and not transitive by referencing to the recent works, such as Bosi and Herden (2012), Cettolin and Riedl (2019), Evren and Ok (2011), Nishimura and Ok (2016), and Ok (2002). Once again, the emphasis of our suggested future works needs to be placed on, instead of their theoretical beauties, how the consequently derived results can be practically employed to produce tangible economic values, such as improved firm performance, through introducing better products or services and through better estimates of consumer behaviors.

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Challenging the Gambler's Fallacy: A Note

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Abstract

Contrary to the gambler's fallacy, we found that past betting outcomes can be predictive. Betting on the outcomes of games in the National Football League following certain streaks against the spread produced wins-to-bets ratios that were statistically significant at five percent.

Background

Streaks of the same betting outcomes can be very tantalizing. Gamblers at a Monte Carlo casino in 1913 were tantalized by the consecutive spins on a roulette wheel that had landed on the color black. Before the streak ended at 26, thousands had been bet futilely on the "overdue" color red. Investors behave in a similar fashion when they liquidate a position that has gone up after a long series of trading sessions because they believe that a decline in the position is overdue. Both are examples of the gambler's fallacy—the mistaken belief that because something has happened less (more) frequently than expected, it is now more (less) likely to occur.

Sports bettors can also provide examples of the gambler's fallacy. A team that beats the point spread in successive games will attract bettors who believe that a loss to the points is overdue. Similarly, a win against the points can seem inevitable after consecutive losses. Studies have generally supported the fallacy. Kochman et al. (2023, under review) reported breakeven results when betting on (against) teams in the National Football League (NFL) that had lost to (beaten) the spread in their previous three games. One exception is Kochman and Gilliam (2011), who found that wagers on college football teams to beat (lose to) the spread following two-game losing (winning) streaks produced an above-average return of nearly 57 percent.

The purpose of this study is to re-examine the predictability of a streak's life against the spread with an innovative ratio of streaks-to-streaks-as-long-or-longer (SSLL). Illustratively, if there had been 100 three-game streaks against the spread in one season of the NFL and 25 streaks that had been longer, a ratio of 80 percent (or 100/125) would have resulted. It could then be said that three-game streaks historically had only a 20-percent chance of continuing. Conversely, if streaks longer than three years had been more numerous than three-game streaks, the chance of a three-game streak continuing vis-à-vis ending would have been greater. By arguing that past betting outcomes are not predictive, the gambler's fallacy is stuck with the assumption that our SSLL ratios would have to be roughly 50 percent.

Methodology and Results

To test that assumption, we first defined a streak as consecutive wins or losses against the spread to create the necessary 50-50 gamble a la roulette. Point spreads adjust for uneven talent—making favored teams and underdogs just as likely to reward bettors. We then applied our SSLL ratio to the 5000+ games in the NFL during the 2010-2019 seasons. Finally, we identified 724 two-game winning streaks against the spread, 672 two-game losing streaks against the spread, 306 three-game winning streaks against the spread, 304 three-game losing streaks against the spread, 136 four-game winning streaks against the spread and 131 four-game losing streaks against the spread during the 2010-2019 seasons of the NFL. The source of our data was Marc Lawrence (2020). The results appear in Table 1.

The greatest likelihood of a streak's termination belonged to four-game losing streaks—a wins-to-bets ratio of 58.8 percent. Other W/B ratios with a greater probability that a streak ends rather than continues were three-game losing streaks (57.9 percent), two-game winning streaks (57.7 percent), three-game winning streaks (56.2 percent) and two-game losing streaks (54.8 percent). When winning and losing streaks were combined by length, W/B ratios were 57.0 percent (three-game streaks), 56.3 percent (two-game streaks) and 54.3 percent (four-game streaks). Four of those W/B ratios were statistically greater than the 52.4-percent breakeven rate[1] at a five-percent level of significance: three-game losing streaks, two-game winning streaks, three-game streaks and two-game streaks. The W/B ratio from bets on four-game losing streaks to end (58.8 percent) was only statistically significant at ten percent.

Conclusion

In sum, our novel SSLL ratios exposed betting strategies that not only challenged the gambler's fallacy but also produced above-average returns. Contrary to the belief that past outcomes are not predictive, we found that streaks of wins and losses against the spread can foretell betting outcomes to the extent that excess returns are possible. Streaks numbering two, three and four games demonstrated that their respective terminations were predictable with varying degrees of success although no relationship between predictability and length of streak was evident. Betting that a streak of wins and losses against the spread will end can make the bettor a net winner while satisfying the human instinct to expect change.

Endnote

1. Customary bets of $11 to win $10 necessitates winning 11 out of 21 bets (or 52.4 percent) to break even.

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Table 1: Streaks-to-streaks-as-long-or-longer in the NFL (2010-2019)

	2-game streaks		3-game streaks		4-game streaks	
	Winning	Losing	Winning	Losing	Winning	Losing
W/B	418/724 (57.7%)	368/672 (54.8%)	172/306 (56.2%)	176/304 (57.9%)	68/136 (50.0%)	77/131 (58.8%)
Combined	786/1396 (56.3%)		348/610 (57.0%)		145/267 (54.3%)	

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